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The Structure of Online Consumer Communication Networks

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Abstract

We analyze the structure of bilateral communication links among consumers in virtual communities by a game-theoretic model of network formation. First, link specificity is incorporated, meaning that the more direct links somebody has to maintain with others, the less she is able to specify her attention per link, so that the value of her links decreases. Second, a distinction is made between the social and informational value from communication, where informational value is transferable via indirect links, whereas social value is not. We characterize the set of pairwise stable structures in the case with only social value to indicate the separate impact of link specificity and demonstrate that it includes a wide range of non-standard architectures under large link specificity and particular combinations of fully connected components under low link specificity. In the case with both social and informational value, the joint effect of link specificity and value transferability is shown to reduce the pairwise stable set to particular fragmented architectures under large link specificity or rather to the complete network under small link specificity.

JEL Classification: A14, C79, D85, M31

Keywords: Consumers, Virtual Communities, Bilateral Communication Links, Social vs. Informational Value, Specificity, Transferability, Network Formation, Game Theory

1 Introduction

Websites such as www.saabnet.com, www.ediets.com/community/, and www.healthboards.com allow a growing number of consumers to easily communicate with like-minded individuals based on shared interests around for example products, consumption activities, or personal conditions. Hence, these communication forums are also increasingly valuable for suppliers, since they are media for word-of-mouth and consumer co-production (e.g., Algesheimer et al. 2005, Dellarocas 2003, Hagel and Armstrong 1997).

Compared to the offline world, consumers in these online communities are relatively flexible to choose their communication partners, since by operating online they are less constrained by geographical distance and by existing social networks, like family structures (Wellman et al. 1996, Van Alstyne and Brynjolfsson 2005). The virtual community literature until now has mainly focused on the question why individuals choose to participate in and contribute to online communities (e.g., Bagozzi and Dholakia 2006, McLure Wasko and Faraj 2005) and disregarded the particulars of these so-called “webs of personal relationships in cyberspace” (Rheingold 2000, p.2). Yet, structures of who communicates with whom are distinguishing empirical phenomena (e.g., Holme et al. 2004, Fisher et al. 2006, Trier 2008) and can determine important outcome variables such as the extent to which value is shared throughout the network and how it is distributed (e.g., Granovetter 2005, Ren et al. 2007).

In the current paper we do study the structure of the bilateral communication links within online consumer communities, to which we therefore refer as Online Consumer Communication Networks (OCCNs). We model their formation as a game-theoretic network formation process in which individuals choose to create and maintain links, only if the participants in the link benefit from doing so, which results in a pairwise stable network structure (Jackson and Wolinsky 1996). Thus, this paper illustrates how to use the rich game-theoretic literature on network formation (e.g., Bala and Goyal 2000, Jackson and Wolinsky 1996) can be used in an applied setting. Recently studied other settings are firm collaboration (e.g., Goyal and Joshi 2003, Belleflamme and Bloch 2004) and crime networks (Calvó-Armengol and Zenou 2004). We demonstrate that our online consumer communication setting is an appealing application area.

We introduce the important distinction between social and informational value as motivations for bilateral exchange decisions. This typology was suggested by the virtual community literature regarding the question why individuals choose to participate in and contribute to such a community as a whole (e.g., Dholakia et al. 2004). Social value is related to the fact that individuals may simply enjoy communicating with others, for example because they find it entertaining or because they feel it enhances their self-worth (e.g., Hennig-Thurau et al. 2004). Informational value refers to the fact that consumers may obtain new valuable knowledge from other consumers when they communicate online. Typically, informational value can be transferred relatively easily to third parties through indirect links, whereas social value is even more personal and therefore hardly transferable (without creating a direct link). This transferability is more prominent in online than in offline communication since information can be more easily forwarded to others (Wellman et al.

1996).

To analyze the underlying structure of OCCNs we develop a model for the formation of links that allows us to understand the relative impact of social and informational member orientation. The model describes how agents benefit and lose from being connected and predicts which stable network structures emerge when agents myopically maximize the resulting payoff value. Herein, we incorporate a combination of two important aspects common to OCCNs that has not been investigated before.

First, our model features link specificity in the sense that the more direct connections an individual has to maintain with other individuals, the less she is able to specify her attention per link. Therefore, her additive value per link for others declines and she also derives less additive value from each link with others (Currarini 2007, Jackson and Wolinsky 1996). We assume that two connected agents contribute to their bilateral process of communication value creation according to a standard production function. Higher link specificity implies higher output elasticities in each bilateral value production process and therefore lower advantage of being connected with several agents. Unit output elasticities are adopted to model high link specificity whereas constant returns to scale (i.e., both output elasticities equal $\frac{1}{2}$) reflect low link specificity.

Second, we realize that when the value derived from communication is not only social but also contains an informational element, this is transferable via indirect links (Bala and Goyal 2000). We therefore assume that informational value is not only experienced from direct neighbors, but flows via any path consisting of bilateral communication links connecting two agents.

More specifically, we first deal with the case of communication having social value only (Section 2) in order to illustrate the separate impact of link specificity on network structure. When link specificity is high, the set of pairwise stable structures is characterized by two simple conditions and is shown to contain a wide range of non-standard architectures, including highly connected and “small world” structures, whereas previous models for social and economic network formation mostly predicted simple architectures like stars and wheels. When link specificity is low, particular combinations of fully connected components are pairwise stable, similar to the prediction of the co-author model of Jackson and Wolinsky (1996).

Next, we deal with the case of communication from which both social and informational value is derived (Section 3) in order to illustrate the impact of value transferability on structure. Under high link specificity, only structures that consist of disjoint star components of two or three agents are shown to be pairwise stable. Apparently, the combination of these two features: high link specificity, which is an example of a negative network externality, and even marginal informational value transferability, which is an example of a positive network externality (Asvanund et al. 2004), has a strong fragmentizing effect on the emerging pairwise stable network structures. Under low link specificity, the opposite effect takes place: already with small informational value transferability, only the complete network structure is pairwise stable.

Subsequently, Section 4 concludes and offers directions for further research.

2 Social value

Since the communication structure of an Online Consumer Communication Network (OCCN) determines value for participants and indirectly also for suppliers, we capture its formation in a game-theoretical model. Although we believe that OCCNs typically combine social and informational value aspects in their communication, we first deal with the simpler case in which only social value is derived from communication. This approach allows us to illustrate the separate impact of link specificity on communication structure.

Link specificity (Currarini 2007, Jackson and Wolinsky 1996) means that the more direct connections an individual has to maintain with other individuals, the less she is able to specify her attention per link. Therefore, her additive value per link for others declines and she also derives less additive value from each link with others. These negative externalities of link formation are crucial in our communication context, since here no benefits arise from individual contributions as such. The reason is that communication is only valuable if it is two-sided, thus effort has to be invested by both sender and receiver.¹

In short, the objective of this section is to develop a model for network formation in OCCNs with only social value from communication. We use the concept of pairwise stability to characterize the category of stable network structures.²

2.1 Model and stability concept

An OCCN is described by (N, g) , where $N = \{1, \dots, n\}$, $n \geq 3$, is a community of agents. A direct link $g_{i,j}$ between agents i and j in this community ($i, j \in N; i \neq j$) is interpreted as a virtual communication relationship between i and j which is established if they both wish the link. These relationships are expressed by undirected links: for any two agents i and j , $g_{i,j} = g_{j,i}$. By definition, $g_{i,i} = 0$, as agents do not establish communication links with themselves. In this community agents only derive social value from interaction.

In case of an isolated relationship between two agents, each agent experiences social value $V^s > 0$ as the outcome of their joint communication production process. However, maintenance of the communication relationship costs effort: investment of both agents is needed in order to make the communication specific to their personal circumstances and hence useful. Accordingly, in case of a structure where two agents do not form an isolated pair, both agents are assumed to divide their effort equally among all their relationships, as a result of which, in an extreme case, the potential social communication value is divided proportionally by the number of links that agents face. However, since agents may have economies of scale in coping with several links, the extent of link specificity can be smaller.

¹In contrast, in the co-author setting, which has been the subject of investigation in earlier research (Jackson and Wolinsky 1996), each co-author can write independently as well.

²We do recognize that next to the social value derived from relationships with specific other participants within an OCCN, participants can also derive social value from the community as a whole (cf. Ren et al. 2007). However, since this is not expected to influence the specific linking decisions they make, we assume it to be constant in our model.

We assume that the contributions of two agents in their bilateral process of communication value creation are reflected by a Cobb-Douglas production function with both output elasticities equal to ρ , where $\rho = 1$ will be considered to constitute the case of high link specificity and $\rho = \frac{1}{2}$ coincides with constant returns to scale and will be considered to constitute the case of low link specificity. Therefore, the total payoff for agent i in link structure g is given by

$$(1) \quad \Pi_i(g) = \begin{cases} \sum_{j \in N_i(g)} \frac{V^s}{(\mu_i(g) \cdot \mu_j(g))^\rho} & \text{if } \mu_i(g) > 0 \\ 0 & \text{if } \mu_i(g) = 0, \end{cases}$$

where $g_{i,j}$ indicates with a 1 or a 0 whether i is directly linked to j or not; $N_i(g)$ is the set of agents with whom i has a direct link, where agent j is a neighbor of agent i if $j \in N_i(g)$, and $\mu_i(g) = |N_i(g)|$ is the number of neighbors of agent i , which is also referred to as the degree of i ; $V^s > 0$ denotes the social value that i would derive from communication with j if neither i nor j were linked to any other agent; and $\rho \leq 1$ indicates the level of link specificity.³

For the model thus described we predict which stable network structures emerge by using the concept of pairwise stability (Jackson and Wolinsky 1996), where a network structure is stable if no single agent can strictly improve her payoff by deleting one of her direct links and no pair of agents can both weakly improve their payoffs by creating a direct link while at least one of the two members strictly improves her payoff by doing so. This solution concept is weak in the sense that it only assumes stability against deviations of exactly one link (which involves the permission of two agents in the case of link formation), reflecting a form of myopia. Alternatively, the model could be analyzed by applying the Nash solution (Bala and Goyal 2000), which assumes stability against single-agent deviations of more than one link. Because of the extreme coordination problem of the Nash concept in two-sided link formation and since the weak concept of pairwise stability already clearly and interestingly constrains the number of network structures that are stable, we choose for the pairwise stability solution.

In our notation, we have the following definition.

Definition 1 (pairwise stability) The structure g is pairwise stable if for all $i, j \in N$ with $g_{i,j} = 1$ it holds that

$$\Pi_i(g) \geq \Pi_i(g') \quad \text{and} \quad \Pi_j(g) \geq \Pi_j(g'),$$

where g' is such that $g'_{i,j} = 0$ and $g'_{k,\ell} = g_{k,\ell}$ for all $\{k, \ell\} \neq \{i, j\}$, and for all $i, j \in N$ with $g_{i,j} = 0$ it holds that

$$\begin{aligned} & \Pi_i(g) > \Pi_i(g') \quad \text{or} \\ & \Pi_j(g) > \Pi_j(g') \quad \text{or} \\ & (\Pi_i(g) = \Pi_i(g') \quad \text{and} \quad \Pi_j(g) = \Pi_j(g')), \end{aligned}$$

³For comparison: the payoff function in the co-author model of Jackson and Wolinsky (1996) can be written as $\Pi_i(g) = \sum_{j \in N_i(g)} \left(\frac{V^s}{\mu_i(g)} + \frac{V^s}{\mu_j(g)} + \frac{V^s}{\mu_i(g) \cdot \mu_j(g)} \right)$.

where g' is such that $g'_{i,j} = 1$ and $g'_{k,\ell} = g_{k,\ell}$ for all $\{k, \ell\} \neq \{i, j\}$.

2.2 Stable structures under high link specificity

First, we evaluate pairwise stability under high link specificity, which we obtain by setting $\rho = 1$. We prove that in this case, the class of pairwise stable network structures can be described by two easily verifiable conditions: (i) they are what we call equal neighbor degree structures, meaning that everybody has at least one neighbor and every neighbor of agent i has the same degree, and (ii) there is at most a difference of one between the degrees of agents in the same component.

Definition 2 (equal neighbor degree structure) A structure g is an equal neighbor degree structure when it holds for each agent i that $\mu_i(g) \geq 1$ and for all agents $j, j' \in N_i(g)$ that $\mu_j(g) = \mu_{j'}(g)$. Here we adopt the following notation: the own degree of agent i is denoted by d_i and her neighbors' degree by e_i .

Definition 3 (path) A path in g connecting i and j is a sequence of agents $k_1, \dots, k_m \in N$ for whom it holds that $g_{i,k_1} = g_{k_1,k_2} = \dots = g_{k_{m-1},k_m} = g_{k_m,j} = 1$.

Definition 4 (component) A component c in g is a structure among a set of agents $C \subseteq N$ for whom it holds that for all $i, j \in C, i \neq j$, there exists a path in c connecting i and j , and for any $i \in C$ and $j \in N$, $g_{i,j} = 1$ implies $c_{i,j} = 1$.

Definition 5 (star) A structure g is a star if it has exactly $n - 1$ links and there exists an agent j for whom it holds that $g_{j,i} = 1$ for all $i \neq j$. Similarly, a component c is a star if it has exactly $|C| - 1$ links and it contains an agent j for whom it holds that $g_{j,i} = 1$ for any other $i \in C$. Agent j is called the center agent whereas the other agents are the periphery agents of the star.

Example 6 A structure consisting of star components is an equal neighbor degree structure.

Example 7 The structure given in Figure 1 is an equal neighbor degree structure.

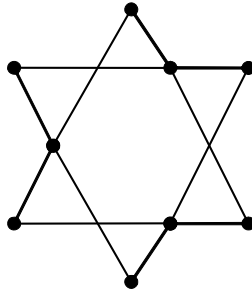


Figure 1: An equal neighbor degree structure

Before providing the main result in Proposition 9, we first derive Lemma 8.

Lemma 8 *When $\rho = 1$, a structure is pairwise stable if and only if it is an equal neighbor degree structure where it holds for each not directly linked pair of agents i, j that*

$$(2) \quad e_i \leq d_j \text{ or } e_j \leq d_i \text{ or } (e_i = d_j + 1 \text{ and } e_j = d_i + 1).$$

Proof. (\Leftarrow) Assume that g is an equal neighbor degree structure where for each not directly linked pair of agents i, j condition (2) is satisfied. The payoff of an agent i as expressed in equation (1) can be written as

$$\Pi_i(g) = \sum_{j \in N_i(g)} \frac{V^s}{\mu_i(g)\mu_j(g)} = d_i \frac{V^s}{d_i e_i} = \frac{V^s}{e_i},$$

so i does not want to delete a link, for then her payoff would reduce to zero if $d_i = 1$, whereas if $d_i > 1$ it would remain equal:

$$(d_i - 1) \frac{V^s}{(d_i - 1)e_i} = \frac{V^s}{e_i}.$$

Moreover, no link between any pair of agents i, j is created if it makes either i or j strictly worse off or both of them equally well off. Therefore, no link is created if

$$(3) \quad \frac{V^s}{e_i} > d_i \frac{V^s}{(d_i + 1)e_i} + \frac{V^s}{(d_i + 1)(d_j + 1)} \quad \text{or}$$

$$(4) \quad \frac{V^s}{e_j} > d_j \frac{V^s}{(d_j + 1)e_j} + \frac{V^s}{(d_i + 1)(d_j + 1)} \quad \text{or}$$

$$(5) \quad \left(\frac{V^s}{e_i} = d_i \frac{V^s}{(d_i + 1)e_i} + \frac{V^s}{(d_i + 1)(d_j + 1)} \text{ and } \frac{V^s}{e_j} = d_j \frac{V^s}{(d_j + 1)e_j} + \frac{V^s}{(d_i + 1)(d_j + 1)} \right).$$

The following shows that $e_i \leq d_j$ implies (3):

$$e_i \leq d_j \implies e_i + d_i(d_j + 1) < (d_i + 1)(d_j + 1) \implies \frac{d_i(d_j + 1) + e_i}{(d_i + 1)(d_j + 1)e_i} < \frac{1}{e_i}.$$

Analogously, it can be shown that $e_j \leq d_i$ implies (4), and $(e_i = d_j + 1)$ and $(e_j = d_i + 1)$ implies (5). Therefore, g is pairwise stable.

(\Rightarrow) Assume that the structure g is pairwise stable. First, suppose that there is an agent i for whom it holds that $\mu_i(g) = 0$. Then her payoff would strictly improve from a link with some other agent k . It is obvious that also k 's payoff would strictly increase if $\mu_k(g) = 0$, which contradicts pairwise stability, so consider the case where $\mu_k(g) \geq 1$. The payoff of k without this link equals

$$\sum_{j \in N_k(g)} \frac{V^s}{\mu_k(g)\mu_j(g)} = \frac{V^s}{\mu_k(g)} \left(\sum_{j \in N_k(g)} \frac{1}{\mu_j(g)} \right),$$

whereas by linking with i it would become

$$\begin{aligned} \sum_{j \in N_k(g)} \frac{V^s}{(\mu_k(g)+1) \cdot \mu_j(g)} + \frac{V^s}{(\mu_k(g)+1) \cdot 1} &= \frac{V^s}{(\mu_k(g)+1)} \left(\sum_{j \in N_k(g)} \frac{1}{\mu_j(g)} + 1 \right) \\ &\geq \frac{V^s}{\mu_k(g)} \left(\sum_{j \in N_k(g)} \frac{1}{\mu_j(g)} \right). \end{aligned}$$

The inequality follows from the observation that the expression before the inequality equals V^s times the average of the terms $1/\mu_j(g)$ and 1, the expression after the inequality is equal to V^s times the average of the terms $1/\mu_j(g)$, and that $1 \geq 1/\mu_j(g)$ for all $j \in N_k(g)$. This contradicts pairwise stability of g . It follows that $\mu_i(g) \geq 1$ for all $i \in N$.

Secondly, suppose that for some i it does not hold that $\mu_j(g)$ is constant for all $j \in N_i(g)$. Then there is an agent $k \in N_i(g)$ such that

$$\mu_k(g) > \frac{\sum_{j \in N_i(g)} \mu_j(g)}{\mu_i(g)}.$$

The payoff for i is given by

$$\sum_{j \in N_i(g)} \frac{V^s}{\mu_i(g) \cdot \mu_j(g)} = \frac{V^s}{\mu_i(g)} \sum_{j \in N_i(g)} \frac{1}{\mu_j(g)},$$

whereas by deleting the link with k , the payoff for i would become

$$\begin{aligned} \sum_{j \in N_i(g)} \frac{V^s}{(\mu_i(g)-1) \cdot \mu_j(g)} - \frac{V^s}{(\mu_i(g)-1) \cdot \mu_k(g)} &= \frac{V^s}{(\mu_i(g)-1)} \left(\sum_{j \in N_i(g)} \frac{1}{\mu_j(g)} - \frac{1}{\mu_k(g)} \right) \\ &> \frac{V^s}{\mu_i(g)} \sum_{j \in N_i(g)} \frac{1}{\mu_j(g)}, \end{aligned}$$

where the last inequality follows immediately from the interpretation of the last two terms as V^s times an average of numbers $1/\mu_j(g)$. This contradicts pairwise stability, so $\mu_j(g) = \mu_{j'}(g)$ for all $j, j' \in N_i(g)$. We have shown that a pairwise stable structure is an equal neighbor degree structure.

Finally, suppose that there exists a not directly linked pair i, j for which condition (2) is not satisfied, implying

$$(6) \quad e_i \geq d_j + 1 \text{ and } e_j \geq d_i + 1 \text{ and } (e_i > d_j + 1 \text{ or } e_j > d_i + 1).$$

Then i and j want to create a link between them, since this would cause the payoff for agent i to become

$$d_i \frac{V^s}{(d_i+1)e_i} + \frac{V^s}{(d_i+1)(d_j+1)} \geq d_i \frac{V^s}{(d_i+1)e_i} + \frac{V^s}{(d_i+1)e_i} = \frac{V^s}{e_i},$$

and for agent j to become

$$d_j \frac{V^s}{(d_j+1)e_j} + \frac{V^s}{(d_j+1)(d_i+1)} \geq d_j \frac{V^s}{(d_j+1)e_j} + \frac{V^s}{(d_j+1)e_j} = \frac{V^s}{e_j},$$

where according to the last condition in (6) at least one of the inequality signs is strict. This contradicts pairwise stability too. Therefore, g is an equal neighbor degree structure with

$$e_i \leq d_j \text{ or } e_j \leq d_i \text{ or } (e_i = d_j + 1 \text{ and } e_j = d_i + 1)$$

for each not directly linked pair of agents i, j . ■

Condition (2) in Lemma 8 can be further simplified, leading to the following proposition.

Proposition 9 *When $\rho = 1$, a structure is pairwise stable if and only if it is an equal neighbor degree structure where it holds for each pair of agents k, ℓ in the same component that*

$$(7) \quad |d_k - d_\ell| \leq 1.$$

Proof. Considering Lemma 8, it is sufficient to show that in an equal neighbor degree structure condition (2) holds for each not directly linked pair i, j if and only if condition (7) is satisfied for each pair k, ℓ in the same component.

(\Leftarrow) Assume an equal neighbor degree structure where for each pair k, ℓ in the same component condition (7) is satisfied. Let i, j be any not directly linked pair. If $e_i \leq d_j$, condition (2) is satisfied. If not, then $e_i > d_j$ and we can derive by applying condition (7) twice that

$$e_j \leq d_j + 1 \leq e_i \leq d_i + 1.$$

If $e_j \leq d_i$, condition (2) is satisfied. If not, then $e_j = d_i + 1$ and condition (2) is satisfied if it also holds that $e_i = d_j + 1$. Suppose not, then $e_i \geq d_j + 2$ and we can derive by applying condition (7) that

$$e_i \geq d_j + 2 \geq (e_j - 1) + 2 = d_i + 2,$$

which contradicts condition (7). Therefore, condition (2) is satisfied.

(\Rightarrow) Assume an equal neighbor degree structure where for each not directly linked pair i, j condition (2) is satisfied. Let k, ℓ be any pair in the same component, so there exists at least one path between k and ℓ . Assume that the total number of agents on any of these paths is odd. Due to the equal neighbor degree structure it holds that $d_k = d_\ell$, so condition (7) is satisfied.

Assume that the total number of agents on all of these paths is even. We consider three cases.

(i) $N_k(g) \setminus \{\ell\} = \emptyset$ and $N_\ell(g) \setminus \{k\} = \emptyset$. It follows that the component consists of k and ℓ only, so condition (7) trivially holds.

(ii) $N_k(g) \setminus \{\ell\} \neq \emptyset$ and $N_\ell(g) \setminus \{k\} \neq \emptyset$. Consider $m \in N_k(g) \setminus \{\ell\}$. Due to the equal neighbor degree structure it holds that

$$d_k = e_m = e_\ell \text{ and } e_k = d_m = d_\ell.$$

Since ℓ and m are not directly linked, by condition (2) we have

$$d_k = e_\ell \leq d_m = d_\ell \text{ or } d_k = e_m \leq d_\ell \text{ or } (d_k = e_\ell = d_m + 1 = d_\ell + 1 \text{ and } d_k = e_m = d_\ell + 1),$$

so $d_k \leq d_\ell + 1$. By the same argument, using some $n \in N_\ell(g) \setminus \{k\}$, we find $d_\ell \leq d_k + 1$. Consequently, condition (7) is satisfied.

(iii) (Without loss of generality) $N_k(g) \setminus \{\ell\} = \emptyset$ and $N_\ell(g) \setminus \{k\} \neq \emptyset$. Since k is connected to ℓ , we have $N_k(g) = \{\ell\}$, $d_k = 1$, $k \in N_\ell(g)$, and $d_\ell \geq 2$. As in case (ii), using some $m \in N_\ell(g) \setminus \{k\}$, it follows that $d_\ell \leq d_k + 1 = 2$. Therefore, it holds that $d_\ell = 2$. Due to the equal neighbor degree structure we find $d_m = d_k = 1$. We have shown that g is a three-agent star. Clearly, condition (7) holds. ■

The following examples illustrate the wide range of structures thus proven to be pairwise stable in the social value case.

Definition 10 (complete structure) A structure g is complete if all agents are connected, so for all $i, j \in N$ it holds that $g_{i,j} = 1$.

Definition 11 (wheel structure) A structure g is a wheel if it has exactly n links and there exists a sequence of different agents $k_1, \dots, k_n \in N$ for whom it holds that $g_{k_1, k_2} = g_{k_2, k_3} = \dots = g_{k_{n-1}, k_n} = g_{k_n, k_1} = 1$.

Definition 12 (regular structure) A structure g is regular if it exists of one component and for each agent $i \in N$ it holds that $d_i = d$.

Corollary 13 *When $\rho = 1$, the complete, wheel, or any regular structure is pairwise stable, for it is an equal neighbor degree structure where it holds for each pair of agents k, ℓ in the single component that*

$$|d_k - d_\ell| = 0 \leq 1.$$

Example 14 A non-regular structure that is pairwise stable under $\rho = 1$ is given in Figure 2.

Example 15 A structure consisting of multiple components that is pairwise stable under $\rho = 1$ is given in Figure 3.

Example 16 A “small world” is a structure with local clusters of highly interlinked agents together with agents that link the various clusters. As a consequence, although most agents are not directly connected, every agent is indirectly linked to every other agent by a relatively small number of steps. A “small world” structure that is pairwise stable under $\rho = 1$ is given in Figure 4.

Note that this wide set of stable structures includes complex real-life architectures (e.g., Dodds et al. 2003), whereas previous models for social and economic network formation mostly predicted simple architectures like stars and wheels (e.g., Bala and Goyal 2000, Goyal and Vega-Redondo 2007).

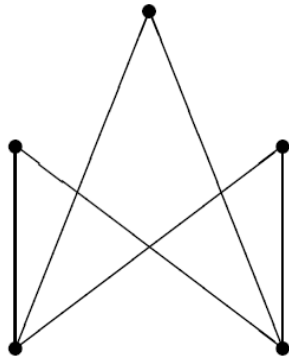


Figure 2: A non-regular pairwise stable structure

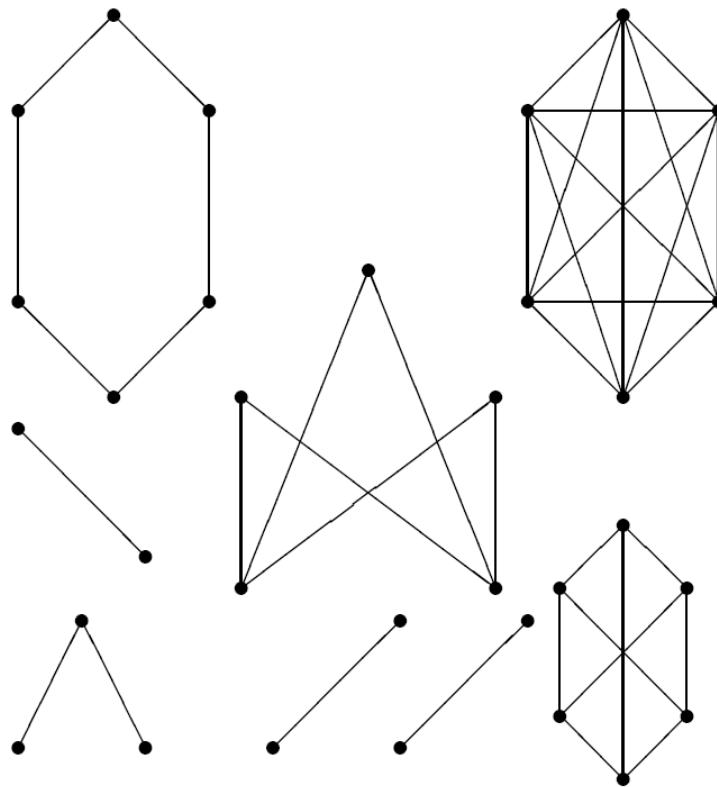


Figure 3: A multiple-component pairwise stable structure

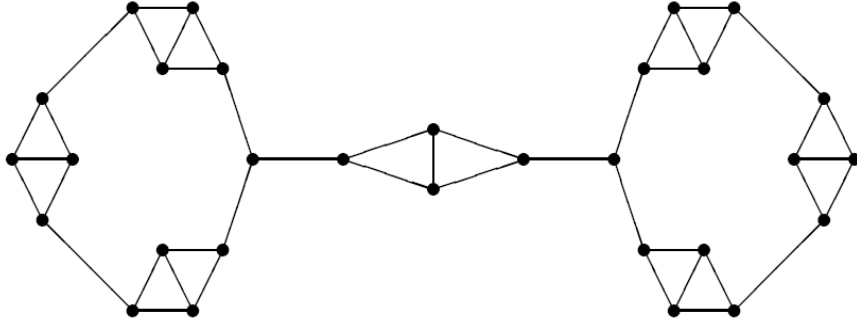


Figure 4: A “small world” pairwise stable structure

2.3 Stable structures under low link specificity

For low link specificity obtained by setting $\rho = \frac{1}{2}$, we show that particular combinations of fully connected components are pairwise stable, similar to the prediction of the co-author model of Jackson and Wolinsky (1996) that a pairwise stable structure can be partitioned into fully intraconnected components, each of which has a different number of members: if m_{c_1} is the number of members of one such component and m_{c_2} is the next largest size, then $m_{c_1} > (m_{c_2})^2$.

Proposition 17 *When $\rho = \frac{1}{2}$, a structure consisting of fully connected components, each of which has a different number of members: if m_{c_1} is the number of members of one such component and $m_{c_2} > 1$ is the next largest size, then $m_{c_1} \geq 4m_{c_2} - 2$, is pairwise stable.*

Proof. Assume that structure g consists of fully connected components of different size. Let m_{c_1} be the number of agents in such a component and m_{c_2} in the next largest one. No member of either of these components wants to delete a link, for the current payoff for such an agent is V^s , whereas deleting a link would reduce it to 0 when $m_c = 2$ or to

$$\frac{m_c - 2}{\sqrt{(m_c - 2)(m_c - 1)}} V^s$$

when $m_c \geq 3$. Let i be an agent in the component with m_{c_2} members and k an agent in the m_{c_1} -sized component. Creating a link with k would change i 's payoff to

$$\left(\frac{m_{c_2} - 1}{\sqrt{(m_{c_2} - 1)m_{c_2}}} + \frac{1}{\sqrt{m_{c_1}m_{c_2}}} \right) V^s,$$

which is not more than her current payoff V^s when

$$m_{c_1} \geq \frac{1}{(\sqrt{m_{c_2}} - \sqrt{m_{c_2} - 1})^2},$$

which can be rewritten as $m_{c_1} \geq 4m_{c_2} - 2$ since m_{c_1} and m_{c_2} are integers. Therefore, under this condition g is pairwise stable. ■

Notice that the set of pairwise stable structures described in Proposition 17 includes the category of pairwise stable structures in the co-author model of Jackson and Wolinsky (1996) (when $n \neq 7$), whereas it is included in the category of pairwise stable structures under high link specificity ($\rho = 1$).

3 Informational as well as social value

This section introduces the case in which both social and informational value is derived from communication in OCCNs. Thus, we can illustrate the impact of value transferability on communication structure along with the effect of link specificity. Value transferability (Bala and Goyal 2000) means that value from communication is not only derived by direct neighbors, but can also be transferred via indirect links. More specifically, we make a distinction between social and informational value derived from communication, where only informational value is transferable through the network. For example, social value from communication between two Saab enthusiasts only exists for the two communication partners, but informational value (e.g., from a solution to a technical problem) can exist for others in the network. After proposing a model for network formation in this setting, the pairwise stable network structures are characterized again. We show that the set of stable structures is much more limited in range than in the purely social value setting.

3.1 Model

An OCCN is described by (N, g) , where $N = \{1, \dots, n\}$, $n \geq 3$, is a community of agents. A direct link $g_{i,j}$ between agents i and j in this community ($i, j \in N; i \neq j$) can be interpreted as a virtual communication relationship between i and j which is established if they both wish the link. These relationships are expressed by undirected links: for any two agents i and j , $g_{i,j} = g_{j,i}$, and $g_{i,i} = 0$.

In case of an isolated relationship between two agents where interaction only has social value, each agent experiences social value $V^s > 0$ as the outcome of their joint communication production process. In case of an isolated relationship between two agents where interaction only has informational value, each agent experiences informational value $V^i > 0$ as the outcome of their joint communication production process. In general, agents are assumed to give relative attention to informational and social value in the proportions α and $1 - \alpha$ respectively, where α is assumed to be constant satisfying $0 < \alpha \leq 1$.

Again we assume that the contributions of two agents in their bilateral process of communication value creation are reflected by a Cobb-Douglas production function with both output elasticities equal to ρ , where $\rho = 1$ will be considered to constitute the case of high link specificity and $\rho = \frac{1}{2}$ coincides with constant returns to scale and will be considered to constitute the case of low link specificity.

Moreover, informational value is, without any decay except for this effort division, transferred to third parties through indirect links (paths of links), whereas social value is not transferable. This is due to the fact that in the direct communication production process of two agents, any of them can use the informational value that she acquired during the bilateral communication creation with other neighbors. Consequently, agent j_0 experiences not only first-step informational payoff from her direct neighbors:

$$\Pi_{j_0}^{1i}(g) = \sum_{j_1 \in N_{j_0}(g)} \frac{V^i}{(\mu_{j_0}(g) \cdot \mu_{j_1}(g))^p},$$

which is similar to the social payoff in equation (1), but also second-step informational payoff:

$$\Pi_{j_0}^{2i}(g) = \sum_{j_1 \in N_{j_0}(g)} \frac{1}{(\mu_{j_0}(g) \cdot \mu_{j_1}(g))^p} \sum_{j_2 \in N_{j_1}(g) \setminus \{j_0\}} \frac{V^i}{(\mu_{j_1}(g) \cdot \mu_{j_2}(g))^p},$$

third-step informational payoff:

$$\Pi_{j_0}^{3i}(g) = \sum_{j_1 \in N_{j_0}(g)} \frac{1}{(\mu_{j_0}(g) \cdot \mu_{j_1}(g))^p} \sum_{j_2 \in N_{j_1}(g) \setminus \{j_0\}} \frac{1}{(\mu_{j_1}(g) \cdot \mu_{j_2}(g))^p} \sum_{j_3 \in N_{j_2}(g) \setminus \{j_1, j_0\}} \frac{V^i}{(\mu_{j_2}(g) \cdot \mu_{j_3}(g))^p},$$

and so forth, thus the overall informational payoff for agent j_0 is equal to

$$\begin{aligned} \Pi_{j_0}^i(g) &= \sum_{q=1}^{n-1} \Pi_{j_0}^{qi}(g) \\ &= V^i \sum_{q=1}^{n-1} \prod_{r=1}^q \sum_{j_r \in N_{j_{r-1}}(g) \setminus \{j_{r-2}, j_{r-3}, \dots, j_0\}} \frac{1}{(\mu_{j_{r-1}}(g) \cdot \mu_{j_r}(g))^p} \\ &= \sum_{q=1}^{n-1} \sum_{r=1}^q \sum_{j_r \in N_{j_{r-1}}(g) \setminus \{j_{r-2}, j_{r-3}, \dots, j_0\}} \frac{V^i}{(\mu_{j_0}(g) \cdot \prod_{b=1}^{q-1} (\mu_{j_b}(g))^2 \cdot \mu_{j_q}(g))^p}. \end{aligned}$$

Therefore, the total payoff for agent i in link structure g is given by

$$(8) \quad \Pi_i(g) = \begin{cases} \alpha \sum_{j \in \bar{N}_i(g)} \sum_{p \in \mathcal{P}_{i,j}(g)} \frac{V^i}{(\mu_i(g) \cdot \prod_{k \in \check{p}} (\mu_k(g))^2 \cdot \mu_j(g))^p} & \text{if } \mu_i(g) > 0 \\ + (1 - \alpha) \sum_{j \in N_i(g)} \frac{V^s}{(\mu_i(g) \cdot \mu_j(g))^p} & \\ 0 & \text{if } \mu_i(g) = 0, \end{cases}$$

where α is the proportion of communication through each link in the community that concerns product-, service- or firm-related information and $1 - \alpha$ is the proportion of communication through each link in the community that concerns social interaction; $\bar{N}_i(g)$ is the set of agents with whom i has either a direct or an indirect link; $\mathcal{P}_{i,j}(g)$ is the set of paths between i and j , and \check{p} is the set of agents on path p between i and j ; and $V^i > 0$ denotes the informational value that i would derive from communication with j if neither i nor j were linked to any other agent and interaction would only have informational value, and $V^s > 0$ denotes the social value that i would derive from communication with j if neither i nor j were linked to any other agent and interaction would only

have social value.

For the model thus described we again use the concept of pairwise stability (Jackson and Wolinsky 1996) to predict which network structures are stable.

3.2 Stable structures under high link specificity

For $\rho = 1$ and $0 < \alpha < 1$,⁴ it can be proven that the pairwise stable structures consist of small star components. First consider the following lemma in which it is shown that the star structure becomes unstable when there are more than three agents.

Lemma 18 *When $\rho = 1$ and $0 < \alpha < 1$, the star structure is pairwise stable if and only if $n = 3$.*

Proof. From the star structure, it is not beneficial for any of the periphery agents to delete her link with the center agent as then her payoff will be zero. For the center agent, deleting a link with any of the periphery agents will provide her with the same payoff. To verify this result, it is crucial to observe that the center agent is not involved in any indirect links to other agents. Periphery agent i does not create a link with another periphery agent i' if and only if this would not decrease her payoff:

$$\begin{aligned} & \alpha V^i \left(\frac{1}{n-1} + \frac{n-2}{(n-1)^2} \right) + (1 - \alpha) V^s \frac{1}{n-1} \geq \\ & \alpha V^i \left(\underbrace{\frac{1}{2(n-1)} + \frac{1}{8(n-1)}}_a + \underbrace{\frac{1}{4} + \frac{1}{4(n-1)^2}}_b + \underbrace{\frac{n-3}{2(n-1)^2} + \frac{n-3}{8(n-1)^2}}_c \right) + (1 - \alpha) V^s \left(\frac{1}{2(n-1)} + \frac{1}{4} \right) \\ & \iff \alpha V^i (4 - n) + (1 - \alpha) V^s (3 - n) \geq 0 \iff n \leq 3, \end{aligned}$$

where the informational payoff elements on the righthandside of the first inequality are derived from (a) the center agent, (b) agent i' , and (c) the other periphery agents consecutively, as a link between i and i' would create a cycle. Since we assumed societies to consist of at least three agents, it holds that $n = 3$. ■

Now the following proposition can be proven.

Proposition 19 *When $\rho = 1$ and $0 < \alpha < 1$, a structure is pairwise stable if and only if it consists of disjoint star components of two or three agents.*

Proof. (\Leftarrow) It is not beneficial for any of the periphery agents in a star component to delete her single link as then her payoff will be zero. Equivalently, for the center agent in a three-agent component, deleting a link with any of the two periphery agents is not beneficial as it will provide her with the same payoff.

⁴The results in the case where the value derived from communication is only informational ($\alpha = 1$) slightly differ from those in this mixed case ($0 < \alpha < 1$). Specifically, it appears that structures also containing one four-agent star component can be pairwise stable.

Link creation between the periphery agents of one three-agent star is eliminated by Lemma 18. Therefore, we only have to examine the following cases (a) – (f):

	pair agent	center agent of 3-agent star	periphery agent of 3-agent star
pair agent	(a)	(b)	(c)
center agent of 3-agent star	x	(d)	(e)
periphery agent of 3-agent star	x	x	(f)

For each of these cases, it can be proven by evaluating the payoffs with and without the link that no link is created: after forming a link in case (a), a pair agent would get payoff:

$$\alpha V^i \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} \right) + (1 - \alpha) V^s \left(\frac{1}{2} + \frac{1}{4} \right) \leq \alpha V^i + (1 - \alpha) V^s,$$

after forming a link in case (b), the pair agent would get payoff:

$$\alpha V^i \left(\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \frac{1}{18} \right) + (1 - \alpha) V^s \left(\frac{1}{2} + \frac{1}{6} \right) < \alpha V^i + (1 - \alpha) V^s,$$

after forming a link in case (c), the pair agent would get payoff:

$$\alpha V^i \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{16} + \frac{1}{32} \right) + (1 - \alpha) V^s \left(\frac{1}{2} + \frac{1}{4} \right) < \alpha V^i + (1 - \alpha) V^s,$$

after forming a link in case (d), a center agent would get payoff:

$$\begin{aligned} & \alpha V^i \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{27} \right) + (1 - \alpha) V^s \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{9} \right) \\ & \leq \alpha V^i \left(\frac{1}{2} + \frac{1}{2} \right) + (1 - \alpha) V^s \left(\frac{1}{2} + \frac{1}{2} \right), \end{aligned}$$

after forming a link in case (e), the center agent would get payoff:

$$\begin{aligned} & \alpha V^i \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{24} + \frac{1}{48} \right) + (1 - \alpha) V^s \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{6} \right) \\ & < \alpha V^i \left(\frac{1}{2} + \frac{1}{2} \right) + (1 - \alpha) V^s \left(\frac{1}{2} + \frac{1}{2} \right), \end{aligned}$$

and after forming a link in case (f), a periphery agent would get payoff

$$\begin{aligned} & \alpha V^i \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} \right) + (1 - \alpha) V^s \left(\frac{1}{4} + \frac{1}{4} \right) \\ & \leq \alpha V^i \left(\frac{1}{2} + \frac{1}{4} \right) + (1 - \alpha) V^s \frac{1}{2}. \end{aligned}$$

(\implies) For this part of the proof, we need some extra notation. The payoff function in (8) can be rewritten as

$$\Pi_i(g) = \frac{1}{\mu_i(g)} \sum_{j \in N_i(g)} T_{i,j}(g),$$

where $T_{i,j}(g)$ is the total payoff that j transmits to i via her direct link with i . Formally,

$$T_{i,j}(g) = \alpha \left(\frac{V^i}{\mu_j(g)} + \sum_{(j' \in \bar{N}_j(g) \setminus \{i\})} \sum_{(p \in \mathcal{P}_{j,j'}(g): i \notin \bar{p})} \frac{V^i}{\mu_{j'}(g) \cdot (\mu_j(g))^2 \cdot \prod_{k \in \bar{p}} (\mu_k(g))^2} \right) + (1 - \alpha) \frac{V^s}{\mu_j(g)}.$$

Assume that g is a pairwise stable structure. Let i be an agent in g and $k \in N_i(g)$ be such that

$$T_{i,k}(g) = \min_{j \in N_i(g)} T_{i,j}(g).$$

Suppose that there exists an agent $\ell \in N_i(g)$ for whom it holds that

$$T_{i,\ell}(g) > T_{i,k}(g).$$

Deleting the link between i and k results in structure g' , where it holds that

$$T_{i,j}(g') \geq T_{i,j}(g), \quad \forall j \in N_i(g'),$$

since k , to whom j might be (in)directly linked, has one costly direct link less, so more informational value might flow from j to i via k . The payoff for i then becomes

$$\Pi_i(g') = \frac{1}{\mu_i(g)-1} \sum_{j \in N_i(g')} T_{i,j}(g') > \frac{1}{\mu_i(g)} \sum_{j \in N_i(g)} T_{i,j}(g) = \Pi_i(g),$$

which contradicts pairwise stability of g . It follows that

$$(9) \quad T_{i,j}(g) = T_{i,j'}(g), \quad \forall j, j' \in N_i(g).$$

Next, suppose that g contains a cycle, meaning that there exists a sequence of agents $k_1, \dots, k_n \in N$ for whom it holds that $g_{k_1, k_2} = g_{k_2, k_3} = \dots = g_{k_{n-1}, k_n} = g_{k_n, k_1} = 1$. Let i be an agent in this cycle. Deleting the link with one of i 's neighbors in the cycle, say k , results in g' , where it holds for the other neighbor of i in the cycle, say m , that

$$T_{i,m}(g') > T_{i,m}(g),$$

since k , to whom m is (in)directly linked, has one costly direct link less, so more informational value flows from k to i via m . Moreover,

$$T_{i,j}(g') \geq T_{i,j}(g), \quad \forall j \in N_i(g').$$

The payoff for i then becomes

$$\begin{aligned} \Pi_i(g') &= \frac{1}{\mu_i(g')-1} \sum_{j \in N_i(g')} T_{i,j}(g') > \frac{1}{\mu_i(g)-1} \sum_{j \in N_i(g')} T_{i,j}(g) \\ &= \frac{1}{\mu_i(g)} \sum_{j \in N_i(g)} T_{i,j}(g) = \Pi_i(g), \end{aligned}$$

where the second equality follows from equation (9). This implies that g is not pairwise stable, leading to a contradiction. We have therefore shown that g does not contain any cycle.

Suppose that g consists of components that are not stars. Since we have already shown that g contains no cycles, all components of g are trees. In a tree the number of links is one less than the number of agents. Moreover, in a tree there is a unique path between any two agents. A tree that is not a star contains an agent, say i , with a neighbor h that only has i as a neighbor, and, moreover, i is directly linked to an agent j who has another neighbor different from i . According to equation (9) it holds that

$$T_{i,h}(g) = T_{i,j}(g).$$

Since h has only one neighbor, i , it follows that

$$T_{i,h}(g) = \alpha V^i + (1 - \alpha)V^s.$$

We now evaluate $T_{i,j}(g)$ and show it is smaller than $T_{i,h}(g)$. Think of $\bar{N}_h(g)$ as a tree with h as top agent. For players $k, k' \in \bar{N}_h(g)$, $k \neq k'$, player k' is a subordinate of k , denoted $k' \in \bar{S}(k)$, if k is on the unique path from h to k' . Player k' is a direct subordinate of k , denoted $k' \in S(k)$, if k' is a subordinate of k and there is a link between k and k' . We write

$$T_{i,j}(g) = \alpha T_{i,j}^i(g) + (1 - \alpha)T_{i,j}^s(g),$$

where

$$(10) \quad T_{i,j}^s(g) = \frac{V^s}{\mu_j(g)} \leq \frac{V^s}{2},$$

and

$$T_{i,j}^i(g) = \frac{V^i}{\mu_j(g)} + \sum_{k \in \bar{S}(j)} \frac{V^i}{\mu_k(g)(\mu_j(g))^2 \prod_{k' \in \bar{p}_{j,k}} (\mu_{k'}(g))^2},$$

where $p_{j,k}$ is the unique path between j and k .

Consider $k \in \bar{S}(i)$. We define the informational payoff that k receives from its subordinates by

$$U_k^i(g) = \frac{1}{\mu_k(g)} \sum_{k' \in S(k)} T_{k,k'}^i(g),$$

where $T_{k,k'}^i(g)$ is defined analogously to $T_{i,j}^i(g)$. We obtain a recursive relation by observing that

$$T_{k,k'}^i(g) = \frac{V^i + U_{k'}^i(g)}{\mu_{k'}(g)}.$$

We show by induction that

$$(11) \quad U_k^i(g) \leq V^i(\mu_k(g) - 1),$$

from which it follows that

$$T_{k,k'}^i(g) \leq \frac{V^i + V^i(\mu_{k'}(g) - 1)}{\mu_{k'}(g)} = V^i,$$

and, consequently,

$$(12) \quad T_{i,j}^i(g) \leq V^i.$$

Let K^0 be the set of agents without subordinates. For $m \geq 1$, let K^m be the set of agents with all subordinates in $K^0 \cup \dots \cup K^{m-1}$. Let m' be the smallest integer for which $j \in K^{m'}$. First consider an agent k in K^0 , the set of agents without subordinates. Then $U_k^i(g) = 0 = V^i(\mu_k(g) - 1)$, so (11) is satisfied.

Suppose that (11) holds for agents in K^m , $m < m'$. Consider an agent $k \in K^{m+1}$.

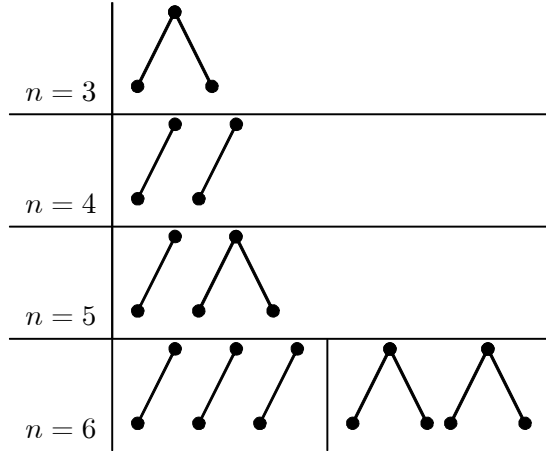
$$\begin{aligned} U_k^i(g) &= \frac{1}{\mu_k(g)} \sum_{k' \in S(k)} T_{k,k'}^i(g) \leq \frac{1}{\mu_k(g)} \sum_{k' \in S(k)} \left(\frac{V^i + V^i(\mu_{k'}(g) - 1)}{\mu_{k'}(g)} \right) \\ &= \frac{\mu_k(g) - 1}{\mu_k(g)} V^i \leq \frac{1}{2} V^i(\mu_k(g) - 1), \end{aligned}$$

so (11) holds for all $k \in \bar{S}(i)$.

Combining (10) and (12) implies $T_{i,j}(g) < T_{i,h}(g)$, a contradiction to equation (9), so g consists of star components only.

The proof of Lemma 18 implies that these stars have at most three agents. “Stars” of a single agent cannot be part of g , for it is always strictly beneficial for this single agent to create a link to the center agent of another star, whereas this center agent is indifferent or improves if she is isolated too. ■

The following table pictures all structures thus proven to be pairwise stable in the case with both social and informational value from communication and $\rho = 1$ for $n \leq 6$.



Comparing these results to the purely social value case, clearly a much smaller range of very fragmented structures turns out to be pairwise stable in the mixed case where transferable informational value also plays a role. Specifically, even with α slightly above zero, regular structures are never pairwise stable and also the example structures in Figures 2, 3, and 4 are not stable anymore. This may seem counter-intuitive, since apparently transferability of informational value causes structures to become more fragmented and therefore *less* able to transfer information. The intuition behind this finding is that the link specificity property of communication is now strong enough to prevent individuals from maintaining many links, because it is strengthened by the transferability of value. For example, in a wheel structure of three agents, an agent cannot improve (or decrease) her social payoff by deleting one of her links, but she can improve her informational payoff:

$$\frac{V^i}{2} + \frac{V^i}{4} > \frac{2V^i}{4} + \frac{2V^i}{16}.$$

The co-author model of Jackson and Wolinsky (1996) also contained a type of link specificity, but since it was not combined with value transferability, the resulting stable structures were not as fragmented. Similarly, the connections model of Jackson and Wolinsky (1996) contained value transferability, but since it was not combined with link specificity, the resulting structures are not fragmented at all. Likewise, most studies reveal less fragmented stable structures, e.g. Goyal & Vega-Redondo (2007) find large star structures in their setting of structural holes. Therefore, our model can explain real-life phenomena like the evolvement of threads in online communities into strong reciprocal ties (Fisher et al. 2006).

3.3 Stable structures under low link specificity

For $\rho = \frac{1}{2}$, it is illustrated that already with small informational value transferability, only the complete network structure is pairwise stable. First we prove the following proposition.

Proposition 20 When $\rho = \frac{1}{2}$ and $\alpha = 1$, the complete structure is pairwise stable.

Proof. The payoff for an agent in the complete structure is

$$1 + \sum_{q=2}^{n-1} \frac{\prod_{r=2}^q (n-r)}{(n-1)^{q-1}},$$

where q indicates the step level, and if she deletes a link it becomes

$$\frac{1+2(n-2)\frac{\sqrt{n-2}}{\sqrt{n-1}}}{n-1} + \sum_{q=3}^{n-1} \frac{\prod_{r=3}^q (n-r)}{(n-1)^{q-1}} \left(1 + \frac{n^2-5n+q+4}{\sqrt{n-1}\sqrt{n-2}}\right).$$

Subtracting the latter from the former gives

$$(13) \quad \sum_{q=3}^{n-1} \left(\frac{\prod_{r=3}^q (n-r)}{(n-1)^{q-1}} \left(n - 3 - \frac{n^2-5n+q+4}{\sqrt{n-1}\sqrt{n-2}} \right) + \frac{2(n-2)-2(n-2)\frac{\sqrt{n-2}}{\sqrt{n-1}}}{(n-1)(n-3)} \right).$$

We have to prove that (13) is nonnegative. Multiplying by $(n-1)$, we find that it is sufficient to show that

$$\sum_{q=3}^{n-1} \left(\left(\prod_{r=3}^q \frac{n-r}{n-1} \right) \left(n - 3 - \frac{n^2-5n+q+4}{\sqrt{n-1}\sqrt{n-2}} \right) + \frac{2(n-2)}{n-3} - \frac{2(n-2)^2}{(n-3)\sqrt{n-1}\sqrt{n-2}} \right) \geq 0.$$

When we define

$$\begin{aligned} a(q) &= \prod_{r=3}^q \frac{n-r}{n-1}, \\ b(q) &= n - 3 - \frac{n^2-5n+q+4}{\sqrt{n-1}\sqrt{n-2}}, \end{aligned}$$

then the first term in (13) is given by

$$\sum_{q=3}^{n-1} a(q)b(q).$$

If this term is nonnegative, then we are done since the second minus the third term in (13) is positive. So suppose the first term is negative.

Notice that $a(q) \geq 0$ and $b(q)$ is decreasing in q , so there is $q \geq 3$ such that $3 \leq q < \bar{q}$ implies $a(q)b(q) \geq 0$ and $\bar{q} \leq q \leq n-1$ implies $a(q)b(q) < 0$. Since

$$\sum_{q=3}^{n-1} a(q)b(q) < 0,$$

we have that

$$\sum_{q=3}^{n-1} a(q)b(q) > \sum_{q=3}^{n-1} \lambda(q)a(q)b(q)$$

for coefficients $\lambda(q)$ greater than or equal to 1 and nondecreasing in q . We define

$$\lambda(q) = \prod_{r=3}^{q-1} \frac{n-1}{n-r},$$

with $\lambda(3) = 1$ by definition. Then our proof is done once we show that

$$\sum_{q=3}^{n-1} \left(\frac{n-q}{n-1} \left(n-3 - \frac{n^2-5n+q+4}{\sqrt{n-1}\sqrt{n-2}} \right) + \frac{2(n-2)}{n-3} - \frac{2(n-2)^2}{(n-3)\sqrt{n-1}\sqrt{n-2}} \right) \geq 0.$$

It holds that

$$\begin{aligned} \sum_{q=3}^{n-1} \frac{n-q}{n-1} (n-3) &= \frac{(n-2)(n-3)^2}{2(n-1)}, \\ \sum_{q=3}^{n-1} \frac{n-q}{n-1} \frac{n^2-5n+4}{\sqrt{n-1}\sqrt{n-2}} &= \frac{(n-2)(n-3)(n-4)}{2\sqrt{n-1}\sqrt{n-2}}, \\ \sum_{q=3}^{n-1} \frac{n-q}{n-1} \frac{q}{\sqrt{n-1}\sqrt{n-2}} &= \frac{n(n-3)(n+2)}{2(n-1)\sqrt{n-1}\sqrt{n-2}} - \frac{2n^3-3n^2+n-30}{6(n-1)\sqrt{n-1}\sqrt{n-2}}, \\ \sum_{q=3}^{n-1} \frac{2(n-2)}{n-3} - \frac{2(n-2)^2}{(n-3)\sqrt{n-1}\sqrt{n-2}} &= 2(n-2) - \frac{2(n-2)^2}{\sqrt{n-1}\sqrt{n-2}}, \end{aligned}$$

where for the third inequality we use the fact that $1^2 + 2^2 + \dots + r^2 = \frac{1}{3}r^3 + \frac{1}{2}r^2 + \frac{1}{6}r$.

After multiplying by 6 and rewriting we obtain the inequality

$$\frac{3n^3-12n^2+27n-30}{n-1} - \frac{3n^4-17n^3+45n^2-73n+54}{(n-1)\sqrt{n-1}\sqrt{n-2}} \geq 0.$$

The expression on the left-hand side is greater than

$$\frac{3n^3-12n^2+27n-30}{n-1} - \frac{3n^4-17n^3+45n^2-73n+54}{(n-1)(n-\frac{8}{5})}.$$

Cross multiplying, we find that the last expression is greater than or equal to zero if and only if

$$3n^4 - 16\frac{4}{5}n^3 + 46\frac{1}{5}n^2 - 73\frac{1}{5}n + 48 \geq 3n^4 - 17n^3 + 45n^2 - 73n + 54.$$

For $n \geq 4$, such is clearly the case. ■

The following example illustrates that already at relatively low α , multi-component structures (cf. Proposition 17 for $\alpha = 0$) are not pairwise stable anymore.

Example 21 Assume $\rho = \frac{1}{2}$ and consider the structure in Figure 5. When $\alpha = 0$, the current payoff for agent i is V^s and if she would create a link with agent k it would become $0.99578 \cdot V^s$. When $\alpha = 1$, the payoff for i is V^i and with a link to k would become $1.65733 \cdot V^i$. When $0 < \alpha < 1$, the payoff for i is $\alpha V^i + (1 - \alpha)V^s$ and with a link to k would become

$$\alpha \cdot 1.65733 \cdot V^i + (1 - \alpha) \cdot 0.99578 \cdot V^s,$$

which is under $V^i = V^s$ larger than the current payoff if $\alpha > 0.0064$. Since k is willing to create a link with i for any α , it holds that this structure in this case is already not pairwise stable anymore for $\alpha > 0.0064$.

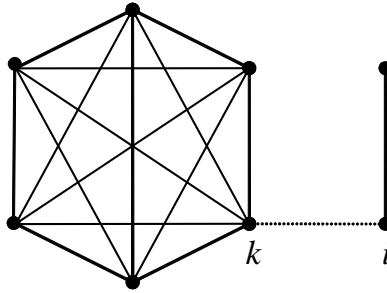


Figure 5: A structure that is pairwise stable when $\rho = \frac{1}{2}$ and $\alpha = 0$

4 Discussion

This paper has shown that the structure of bilateral communication links within OCCNs can be appropriately studied using a model based on the game-theoretic literature of social and economic network formation. A combination of important aspects common to OCCNs was incorporated that had not been investigated until now: the negative externality of link specificity and the positive externality of informational value transferability.

In the case of communication having social value only, illustrating the separate impact of link specificity on structure, the set of pairwise stable structures was characterized for high link specificity and shown to include a wide range of non-standard architectures like highly connected and “small world” structures, whereas previous models for social and economic network formation mostly predicted simple architectures like stars and wheels. For low link specificity, particular combinations of fully connected components were proven to be pairwise stable in line with the co-author model of Jackson and Wolinsky (1996).

In the case of communication from which both social and informational value is derived, illustrating the joint impact of link specificity and value transferability on structure, under high link specificity only structures that consist of disjoint star components of two or three agents were shown to be pairwise stable. Herewith, we predict much more fragmentation than usually in the literature about social and economic network formation, where mostly only either of these two features was included. Under low link specificity, the opposite extreme effect takes place: already with small informational value transferability, only the complete network structure is pairwise stable.

Further research could focus on the welfare properties of the wide variety of structures discussed in this paper. In particular, it can be found that the fragmentation under high link specificity as well as the dense pairwise stable structures under low link specificity are efficient in their own setting. Also, it may be interesting to consider other link specificity values than the cases $\rho = \frac{1}{2}, 1$ studied here. Especially, it can be found that $\frac{1}{2}$ and 1 are indeed suitable polar cases and that for intermediate values the famous tension between stability and efficiency (e.g., Jackson and Wolinsky 1996) is re-established.

Moreover, future research could introduce valuation heterogeneity in the sense that agents represent different values for their fellow customers or have different opinions on the values of their fellow customers (e.g., Galeotti et al. 2006). For example, for $\rho = 1$ and $0 < \alpha < 1$, if we assume a valuation pattern deviating from full homogeneity in the sense that there is one agent j who is valued differently than all other agents, it can be proven that all pairwise stable structures consist of small star components and one possibly larger component without cycles containing the differing agent j but not at the periphery. In particular, this component may be a star component with agent j at the center.

Another extension of the current model could be to relax the assumption that agents divide their available effort equally among all their relationships, thus entering the subject of link quality and dropping the common one-zero formulation of links. As suggested by Goyal (2005), a first step into this direction would be to introduce a distinction between strong links in which both agents actively interact with each other, and weak links in which one agent is active and the other is not, where the passive agent can only access the direct value from her active partner.

Besides, a possible follow-up would be to empirically examine the applicability of the used payoff function in diverse contexts. The model could be tested experimentally, contributing to an emerging literature as surveyed by Kosfeld (2004).

Accordingly, we hope that our current work stimulates future research in the appealing area of OCCNs and the role of balancing social and informational value in these communication networks.

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