Measuring the rebound effect using a stochastic demand frontier approach: the US residential energy demand

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Abstract

This paper brings attention to the fact that the energy demand frontier model introduced by Filippini and Hunt (2011, 2012) is closely connected to the measurement of the so-called rebound effect associated with improvements in energy efficiency. In particular, we show that their model implicitly imposes a zero rebound effect, which contradicts most of the available empirical evidence on this issue. We relax this restrictive assumption by using a convenient reinterpretation of a stochastic frontier model that satisfies the scaling property which is often assumed in production economics. We illustrate our model with an empirical application that aims to estimate a US frontier residential aggregate energy demand function using panel data for 48 states over the period 1995 to 2011.

Keywords: US residential energy demand; efficiency and frontier analysis; state energy efficiency, rebound effect.

JEL Classification: D2, Q4.

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1. Introduction

Reducing energy consumption and emissions is a key policy objective for most governments across the globe and the promotion of energy efficiency policies is seen as a key activity to achieving this goal. In practice, the achievement of savings in energy consumption depends on two issues. First, it is vital that policy makers be able to clearly measure the relative energy efficiency across states and over time. Second, the actual savings in energy consumption might not coincide with the expected savings due to the so-called rebound effect, a phenomenon associated with the consumption of energy and energy services. When the production of an energy service becomes more efficient, then the cost per unit of this service decreases. This cost reduction can produce an increase in the consumption of the energy service that might (at least partially) offset the expected savings in energy consumption derived from the energy efficiency improvements. Measuring the rebound effect is thus crucial in order to properly evaluate the effectiveness of any energy policy instrument that aims to promote energy efficiency improvements.

Regarding the first issue, Filippini and Hunt (2011, 2012) point out that defining and measuring energy efficiency and creating statistical measures as descriptors is a challenging task. They propose the use of a Stochastic Frontier Analysis (SFA) approach to control for characteristics such as the structure of the economy that might bias the usual energy efficiency indicators. These authors illustrate their proposal by estimating an aggregate energy demand frontier model for the total energy consumption of a sample of OECD countries and for the residential energy consumption of the US states. The SFA approach allows them to obtain a "pure" measure of the inefficient use of energy (i.e. 'waste of energy') for each country or state.

Concerning the second issue, there is a large number of empirical studies that use econometric methods to estimate the rebound effect. In their review of the literature, Sorrell and Dimitropoulos (2008) have found a lack of consensus with regard to a consistent method to measure the rebound effect. In principle, it could be *directly* obtained from the elasticity of demand for energy services with respect to changes in energy efficiency. However, relatively few studies follow this approach because data on either energy services or energy efficiency are unavailable or are limited in terms of accuracy. As a consequence the rebound effect is often indirectly measured through the estimate of different elasticities that are considered measures of energy efficiency elasticities of the demand for energy, such as the own-price elasticity of the demand for energy.

The main contribution of this paper is to link the energy demand frontier approach with the estimation of the rebound effect. We first bring attention to the fact that the frontier model introduced by Filippini and Hunt (2011, 2012) also provides a direct measure of the rebound effect. However, we point out that a traditional specification of this model implicitly imposes a zero (or more accurately, constant) rebound effect, which contradicts most of the available empirical evidence on the rebound effect. We next suggest estimating a more comprehensive model to relax the zero rebound effect assumption and examine the compliance with some of the restrictions used in previous studies focused on estimating the rebound effect using econometric techniques. The proposed empirical strategy relies on a convenient reinterpretation of the stochastic frontier model that satisfies the traditional scaling property in production economics. The so-called scaling function is used here as a measure of the rebound effect.

The paper is organized as follows. The next section defines the rebound effect and provides a brief review of the empirical literature on measuring this effect using econometric models. Both standard and extended energy demand frontier models and the econometric specification of our model are introduced in Section 3. The data and results of the estimates are presented in Section 4 with a summary and conclusions in the final section.

2. Measuring the rebound effect: a short review of the empirical literature

The rebound effect is a phenomenon associated with energy consumption. This concept has to do with the idea that an increase in the level of efficiency in the use of energy decreases the marginal cost of supplying a certain energy service and hence may lead to an increase in the consumption of that service. This consumer reaction might therefore partially or totally offset the predicted reduction in energy consumption attributed to energy efficiency improvements using engineering models. Measuring the rebound effect is thus crucial in order to properly evaluate the effectiveness of any energy policy instrument that aims to promote energy efficiency improvements. This issue is particularly relevant for the US residential sector since it accounts for 37% of the national electricity consumption, 17% of greenhouse gas emissions and 22% of primary energy consumption (International Risk Governance Council (IRGC), 2013).

The definition of the rebound effect encompasses different mechanisms that may reduce potential energy savings derived from the improvements in energy efficiency. Frequently, three types of rebound effect are distinguished in the specialized literature. The first one is the *direct* rebound effect, which measures the increase in the use of the product or service that has experienced the efficiency gain. For instance, a homeowner may employ a portion of the energy savings from using an efficient heater to use the heater for longer periods during the winter to warm the house. The second type is the so-called *indirect* rebound effect and measures the reallocation of energy savings to spending on other goods and services that also require energy. For instance, the savings derived from the use of energy-efficient appliances at home can be spent on travel holidays which may lead to an increase in energy consumption and greenhouse gas emissions. The third type is the *economy-wide* rebound effect and captures the structural changes in the economy due to the variation of prices of goods and services as a consequence of energy efficiency improvements. These changes may produce a new equilibrium in the consumption of goods and services (including energy) in the economy.

There is an extensive literature on the concept and measurement of the rebound effect and several approaches have been applied with the aim of quantifying this phenomenon. For instance, in their report for the UK Energy Research Centre, Sorrell and Dimitropoulos (2007) find a wide range of methods that have been applied to measure the direct rebound effect. They identify at least four empirical approaches - single equation models, structural models, discrete/continuous models, and household production models - and several estimation techniques including ordinary least squares, instrumental variables or maximum likelihood. In addition, several empirical strategies have also been used to indirectly measure this rebound effect. An outline of these approaches can be found in Table 1. This table shows three theoretical relationships between two elasticities. The left-hand side elasticity is the energy efficiency elasticity of the demand for energy, which is used to calculate the clearest and most direct measure of the rebound effect (see Saunders, 2000, and Section 3 below). The lack of accurate data on energy services or energy efficiency typically precludes a direct measurement of the rebound effect based on this elasticity, so that its estimation is usually carried out using the right-hand side of the equations in Table 1.

[Insert Table 1 here]

The first empirical approach relies on estimating the energy efficiency elasticity of the demand for energy services or useful work that is often available in personal transportation studies. For this reason, this engineering-based approach is generally used to measure the direct rebound effect associated with travelling by private cars (see for instance Greene *et al.*, 1999b; or Small and Van Dender, 2005). More studies follow the second empirical strategy, based on an estimate of the energy cost elasticity of the demand for useful work. This approach has been advocated by Khazzoom (1980), Greene *et al.* (1999a), Berkhout *et al.* (2000) and Binswanger (2001) and, unlike the first approach, it provides a way to estimate the magnitude of the rebound effect even when the available data provides little or no variation in energy efficiency. However, the validity of this approach relies on the assumption that consumers respond in the same way to decreases in energy prices as they do to improvements in energy efficiency (and vice versa). As Sorrell and Dimitropoulos (2008) pointed out, this assumption is likely to be flawed in many cases. These two approaches require accurate measures of the demand for useful work. This restriction has biased research studies towards personal transportation and household heating, where data about energy services can be easily calculated, e.g., vehicle kilometres in the case of transportation.

It is also possible to estimate the direct rebound effect from the own-price elasticity of the demand for energy, i.e., the third approach. While obtaining measures of useful work can be difficult, data on energy demand is more commonly available. The main advantage of the third approach over previous approaches is that data on either useful work or energy efficiency is not required. This explains why the approach based on the own-price elasticity of the demand for energy is the most popular empirical strategy to measure the rebound effect in other energy commodities or sectors (see for instance Zein-Elabdin, 1997; Berkhout *et al.*, 2000; Roy, 2000 and Bentzen, 2004). However, Sorrell and Dimitropoulos (2008) pointed out that this empirical strategy might also yield biased estimates for the rebound effect if energy efficiency is not explicitly controlled for.[1] In this paper, we propose another approach based on the estimation of an energy demand frontier function. In this approach, the direct rebound effect is estimated from the elasticity of the demand for energy with respect to the level of energy efficiency.

There is a huge variety of estimated rebound effects in the literature not only because different methodological/empirical approaches have been used but also because they have been used to analyse the rebound effect for different energy commodities, sectors, countries or different levels of data aggregation. Since our paper is focused on residential energy demand, we pay attention mainly to the results of papers on household energy demand. Sorrell and Dimitropoulos (2007) find that for household heating the rebound effect usually ranges from 10% to 58% in the short-term and from 1.4% to 60% in the long-term.[2] Household energy demand is dominated by the use of

[1] In particular, this approach relies on the assumption that energy efficiency is unaffected by changes in energy prices.

[2] These rebound effects indicate percentage (expressed in relation to the predicted energy saving) by which the actual energy consumption is larger than the predicted energy consumption after an efficiency improvement. The measuring of the rebound effect is explained in detail in the next section.

fuel and electricity for heating space. Focusing specifically on papers in which the price elasticity of total household electricity demand is estimated, the estimated values suggest an upper bound for the short-term rebound effect in the range of 20% to 35% and between 4% and 225% for the long-term rebound effect. Regarding other household energy services, the reviewed studies suggest a rebound effect for space cooling between 1% and 26%. Other studies produce rather different results. Guertin *et al.* (2003) estimate long-term rebound effects for both water heating and appliances/lighting and obtain values between 32% and 49% and Davis (2007) obtains short-term direct rebound effects for clothes washing lower than 5%.

A recent survey can be found in a report on energy efficiency carried out by the IRGC (2013). This survey is based on the reviews of Greening *et al.* (2000), Sorrell (2007) and Jenkins *et al.* (2011) and summarises the large variety of results obtained from papers that measure rebound effects in the residential sector. This report shows that while for residential lighting there is a narrow range of results of the rebound effect from 5% to 12%, in the rest of energy services there is a wider range of values: for space heating the range goes from 2% to 60%, for space cooling from 0% to 50%, for water heating from less than 10% to 40%, and for other consumer energy services from 0% to 49%. As it can be seen, this more updated survey shows very similar values to the report previously mentioned.

However it should be noted that in our paper we estimate a demand function aggregated at state-level for the US residential energy. Therefore our estimated rebound effect captures an overall effect composed of the sum of direct and indirect effects and hence the ideal lower and upper bounds for our estimates are not entirely clear. The literature has identified large positive as well as negative values for the indirect rebound effect, as found in Thomas and Azevedo (2013) for the household case. There are some papers that exhibit large direct rebound effects, such as Mizobuchi (2008) where a rebound effect of about 27% is found for Japanese households although the effect increases to 115% when capital costs are ignored in the analysis. Indirect rebound effects are usually larger than direct rebound effects and it is less ‘uncommon’ to find indirect rebound effects larger than 100%. Some examples can be found in Lenzen and Dey (2002) with an indirect rebound effect of 123% for Australia, Alfredsson (2004) with an indirect rebound effect up to 300% in Sweden or Brännlund *et al*. (2007) with an indirect rebound effect between 107-115% in CO_2 emissions in a simulation of an efficiency improvement in heating and transport sectors. In some cases this rebound measures can reach extremely large values, as in Druckman *et al.* (2010) who found indirect rebound effects up to 515% for the case of the UK.

3. Measuring rebound effects using an energy demand frontier model

In this section, firstly we summarize the aggregate energy demand frontier model proposed by Filippini and Hunt (2012) to measure the level of “underlying energy efficiency” in the US residential sector. Subsequently, we link this model to the literature on the rebound effect and we introduce a more comprehensive model that allows estimating ‘non-zero’ rebound effects using an SFA approach. Once the econometric specification of the model is presented, we finally discuss new econometric issues that appear when the more general SFA approach is used to estimate rebound effects.

3.1. The standard energy demand frontier model

This approach treats energy as a production factor used in combination with other inputs to produce energy services, and attempts to measure inefficiency in the use of input energy as (positive) deviations from an energy demand frontier function that can be estimated for the whole economy or for a given sector. In general terms, the aggregate energy consumption can be written as follows:

$$q = F(Y, P, X, E, \beta) \cdot e^{v} \quad (1)$$

where q is the aggregate energy consumption, Y is the real income, P is the real energy price, β are parameters to be estimated, and X is a set of control variables such as population, average household size, heating degree days, cooling degree days, the share of detached houses, or time dummy variables. While v is the conventional noise term, E is the level of energy efficiency of a particular state. Since the energy efficiency level is not observed by the researcher, Filippini and Hunt (2012) made use of two assumptions in order to estimate Equation (1). Firstly, they implicitly assumed that the energy demand function is *separable* in the sense that $F(Y, P, X, E, \beta)$ in (1) is decomposed into a function that does not depend on energy efficiency and an energy-efficiency function, that is:

$$F = f(Y, P, X, \beta) \cdot h(E) \quad (2)$$

where $h(E)$ is in turn assumed to be equal to $1/E$. The second assumption is that the unobserved energy efficiency term is bounded (i.e. $0 \leq E \leq 1$). These two assumptions allow using the stochastic frontier approach as the model to be estimated can now be written in logs as:

$$\ln q = \ln f(Y, P, X, \beta) + v + u \quad (3)$$

where $u=-\ln E \geq 0$. The error term in (3) thereby comprises two independent parts. The first part, v, is the classical symmetric random noise, often assumed to be normally distributed, i.e. $v \sim N(0,\sigma_v^2)$. The second part, u, is a one-sided error term capturing the level of underlying energy inefficiency that can vary across states and over time. Following Aigner *et al.* (1977) it is often assumed to follow a half-normal distribution, i.e. $u \sim N^+(0,\sigma_u^2)$. That is, this traditional model (ALS model henceforth) assumes that u is homoscedastic.

Equation (3) is the basic specification of the energy demand frontier that is estimated in Filippini and Hunt (2011, 2012) in order to get state-specific energy efficiency scores.[3] In the case of an aggregate residential energy demand function, $f(Y, P, X, \beta)$ reflects the demand of the residential sector of a state that has *and* uses fully efficient equipment and production processes. If a state is not on the frontier, the distance from the frontier measures the level of energy consumption above the minimum demand of reference, i.e. the level of energy inefficiency. Nevertheless, from an empirical perspective, when using US residential aggregate energy data the aggregate level of energy efficiency of residential appliances is not observed directly[4] and therefore has to be estimated simultaneously with other parameters of the model. To note, that Filippini and Hunt (2012) were more interested in the estimation of an energy demand frontier model using an econometric approach for panel data that consider the problem of unobserved heterogeneity. For this reason they propose the application of

[3] The estimation of (3) can be performed using either cross-sectional or panel data as in Filippini and Hunt (2011, 2012). They also propose to use a relatively simple log-log functional form.
[4] For this reason Filippini and Hunt (2011, 2012) use the expression ‘underlying energy efficiency’.

Mundlak's adjustment to the Pitt and Lee (1981) frontier framework which decreases the bias in inefficiency estimates by separating inefficiency from unobserved heterogeneity. In this paper, the emphasis is rather on the measurement of the rebound effect that on unobserved heterogeneity.[5]

3.2. The (implicit) rebound effect in the standard energy demand frontier model

Although the basic concept of the rebound effect is not controversial, several mathematical definitions of this effect have been employed in the literature according to the availability of price and efficiency data.[6] Here we use the definition mentioned by Saunders (2000) which, in our opinion, provides one of the clearest and most direct measurements of the rebound effect. Following this author, the rebound effect is obtained as:

$$R = 1 + \varepsilon_E \tag{4}$$

where ε_E is the elasticity of energy demand with respect to changes in energy efficiency, i.e. $\varepsilon_E = \partial \ln q / \partial \ln E$. The actual saving in energy consumption will only be equal to the predicted saving from engineering calculations when this elasticity is equal to minus one and hence there is no rebound effect (*R=0*). The rebound effect would be positive (R>0) if actual savings in energy consumption are less than expected, i.e. $-1 < \varepsilon_E$. The rebound effect could be larger than one, i.e. R>1, if improvements in energy efficiency increase energy consumption and hence the elasticity of energy demand with respect to changes in energy efficiency is positive, i.e. $\varepsilon_E > 0$. This somewhat counterintuitive outcome is termed 'backfire' in the literature (Saunders, 1992). In practice, negative rebound effects (R<0) can also be found for some observations if the improvements in energy efficiency produce larger decreases in energy use, i.e. $\varepsilon_E < -1$. Saunders (2008) labelled this - also rather counterintuitive - outcome as 'super-conservation'.[7] Table 2 shows the different rebound effects that we can find in a particular empirical application.

[Insert Table 2 here]

As the one-sided error term in (3) is measuring the level of underlying energy inefficiency, the elasticity of energy demand with respect to changes in energy efficiency is simply $\varepsilon_E = -\partial \ln q / \partial u$. Given the rebound effect definition provided by equation (4), we can then conclude that any energy demand frontier model that includes an inefficiency term as an explanatory variable implicitly provides a *direct* measure of the rebound effect. However, since ε_E in (3) is equal to -1, the standard SFA energy demand frontier model implicitly imposes a zero rebound effect, which contradicts most of the available empirical evidence surveyed in Section 2.

[5] The model that is proposed in the paper can also be estimated including the Mundlak's adjustment but it does not affect to our estimated rebound effects and only implies that some of the estimated coefficients lose significativity. Since this modification seems to be in some way 'redundant' with the approach used here, we have not included this model in the paper. However note that the endogeneity problem that may arise if an energy demand function is estimated without modeling the rebound effect (and it actually depends on price and/or income) might be corrected by using the Mundlak's adjustment.

[6] See, for instance, Sorrell and Dimitropoulos (2008).

[7] For a more extended definition and some examples about this counterintuitive phenomenon see Saunders (2008).

3.3. A frontier energy demand model with non-zero rebound effects

In order to relax the zero rebound effect assumption, we will change the specification of the energy efficiency function $h(E)$ and, in particular, replace $h(E)= E^{-1}$ with the following expression:

$$h(E) = E^{-g[ES(z,\gamma)]} \tag{6}$$

where g is a function that depends on the demand of energy services, ES; z *is* a set of energy services determinants; and γ are new parameters to be estimated. Equation (6) implies that:

$$F = f(Y,P,X,\beta) \cdot E^{-g[ES(z,\gamma)]} \tag{7}$$

and hence the new frontier model to be estimated is:

$$\ln q = \ln f(Y,P,X,\beta) + v + g[ES(z,\gamma)] \cdot u \tag{8}$$

where u=-ln$E$≥*0*.

Several interesting remarks may be made regarding this specification. First, when $g=1$, our model collapses to the basic stochastic frontier demand model used in Filippini and Hunt (2011, 2012). Second, the above model can be viewed as analogous to a traditional heteroscedastic SFA model in production economics that satisfies the scaling property. However, while the (scaling) function g is interpreted in the latter literature as a portion of total estimated inefficiency, we treat the (rebound effect) function g as part of the deterministic frontier.[8] This interpretation allows us to relax the zero rebound effect assumption. Indeed, as we do *not* treat g as part of energy inefficiency, it so happens that the rebound effect is a simple linear transformation of the scaling function, that is:

$$R = 1 + \varepsilon_E = 1 - g(\gamma' z) \tag{9}$$

Given the strong relationship between R and g, we hereafter re-label g as the rebound effect function. Since the rebound effect is linked to changes in the energy required to provide energy services, we assume that g depends on the demand for energy services, *ES.* Like the energy inefficiency level, this variable is not observed by the researcher. This latent variable will be therefore approximated with a set of determinants of the demand for energy services, such as income and energy prices, i.e. $z=(Y,P)$. This seems to be reasonable as most of the literature on the rebound effect associates the rebound effect with energy prices, and the theory often predicts that the rebound effect declines with income.[9] If the rebound effect function does not depend on any covariate, our model simply collapses to the traditional energy demand frontier model that imposes zero (or constant) rebound effects. In contrast, if g varies across

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[8] See Alvarez *et al.* (2006) for a review of the models that incorporate this property in the literature on frontier production functions. In this literature, u is viewed as a measure of "raw" inefficiency, and g is a deterministic function that "adjusts" the "raw" inefficiency level upwards or downwards due to the influence of some potential inefficiency determinants, z. For this reason, g is often referred as the scaling function and *overall* firm inefficiency is a "scale" transformation of some underlying, and unexplained, inefficiency level, u (see Kumbhakar and Lovell, 2000, p. 274).

[9] Wang *et al.* (2012) point out, for instance, that the marginal utility of energy service consumption will decline as household income increases. Thus, energy efficiency improvements may not induce people to consume as much energy services as before. This means that the direct rebound effect might decline with the increase in household income. This is also confirmed in a limited number of studies, e.g. Small and Van Dender (2007) and Wang *et al.* (2012) that have found evidence of a negative relationship between the rebound effect and income.

observations or states, the above equation allows us to get state-specific rebound effects that can be used for further analyses.[10]

Third, as pointed out by Saunders (2008) the choice of a particular functional form will probably condition the estimated magnitude of the rebound effects. In this sense, he suggests using "rebound flexible" functions that can exhibit most of the rebound ranges in Table 2. However, the choice of a particular function is limited by both methodological and practical issues. Indeed, in order to distinguish inefficiency from noise we should impose that $g \geq 0$. Otherwise, the error term $g(\gamma' z) \cdot u$ in equation (8) would no longer have a one-sided distribution and we would not be able to take advantage of the asymmetric distribution of u to decompose the overall error term (i.e. $v+g \cdot u$) into two different stochastic components. Note that in our framework this precludes the existence of rebound effects larger than one or backfire outcomes.

Several tractable specifications of g can be used in a particular empirical application.[11] In this sense, we propose exploring two simple rebound-effect functions (see Table 3). Whereas the RSCFG[12] specification can depict anywhere from zero rebound to super-conservation outcomes (i.e. R<0), the LOGISTIC function precludes this somewhat counter-intuitive outcome as it only takes values between zero and one. The LOGISTIC function is the inverse of the Logit function and coincides with the scaling function introduced by Kumbhakar (1990) in production economics to allow for time-varying efficiency in firms. In addition, this type of function is quite popular in other research fields where the variables to be predicted are probabilities or limited dependent variables are somehow involved in the model. Other specifications have been examined in previous versions of this paper, such as the simple cumulative density function of a standard normal variable, Φ, which like the LOGISTIC function lies between zero and one, or the ratio $\Phi/(1-\Phi)$, which allows for super-conservations outcomes as does the RSCFG model. The results of these models are not shown in the paper as they are very similar to those obtained with the proposed models.

[Insert Table 3 here]

Our reinterpretation of g as a rebound effect opens new issues that do not appear in the standard stochastic production frontier analysis. For instance, equation (8) can be estimated by either assuming $g = \exp(\gamma_0 + \gamma' z)$ and $\sigma_u = 1$ or by assuming that u follows a standard half-normal distribution with $\sigma_u = \exp(\delta_0)$ but where g does not include a separate intercept, i.e. $\gamma_0 = 0$. Only one constant is allowed as they cannot be estimated simultaneously. It is worth mentioning that this issue is not important in production economics as both options yield exactly the same results when modelling

[10] Interesting enough, if z includes income and energy prices, the estimated γ can also be used to test whether both income and price elasticities of energy demand depend on energy efficiency as both elasticities can be respectively written as:

$$\varepsilon_Y = \frac{\partial \ln q}{\partial \ln Y} = \frac{\partial \ln f(Y,P,X,\beta)}{\partial \ln Y} - \frac{\partial g(Y,P,\gamma)}{\partial \ln Y} \cdot \ln E$$

$$\varepsilon_P = \frac{\partial \ln q}{\partial \ln P} = \frac{\partial \ln f(Y,P,X,\beta)}{\partial \ln P} - \frac{\partial g(Y,P,\gamma)}{\partial \ln P} \cdot \ln E$$

[11] Saunders (2008) recommends using extremely comprehensive (flexible) functional forms such as the Gallant and Fourier forms, which can depict the full range of rebound values. These forms are however intractable in our framework as they would interact with the stochastic part of the model and, hence, the maximum likelihood function would be highly non-linear in parameters.

[12] RSCFG comes from Reifschneider and Stevenson (1991), Caudill and Ford (1993) and Caudill, Ford and Gropper (1995). The scaling function introduced by these authors is the most commonly used in the stochastic production frontier literature.

overall firm inefficiency. However, if we estimate an energy demand frontier function, it matters as we would be either magnifying or diminishing the rebound effect.

Whichever option is used, a detail that is important here is that the estimated intercept of g is *biased* because in practice it captures two intercepts: the 'true' intercept of g, and a parameter measuring σ_u. If, for instance, we use an exponential rebound-effect function, the estimated intercept in (8) will be $\hat{\gamma}_0 = \gamma_0 + \delta_0$. A simple empirical strategy is proposed to deal with this uncertainty. This strategy relies on the assumption that our energy inefficiency term u follows the same distribution in both equations (3) and (8), so that the ALS estimate of σ_u can be used to adjust the estimated intercept of g accordingly. For instance, if the rebound effect function is $g = \exp\,(\gamma_0 + \gamma' z)$ and $\hat{\delta}_0^{ALS}$ is used as an estimate of $\hat{\delta}_0$, the estimate of γ_0 is then obtained by adjusting the estimated intercept as follows: $\gamma_0 = \hat{\gamma}_0 - \hat{\delta}_0^{ALS}$. The same strategy is followed in the LOGISTIC specification of the rebound effect function as it also cannot be estimated with two intercepts.

4. Data and results

Our empirical application is based on a balanced US panel data set for a sample of 48 states over the period 1995 to 2011. That is, we have added four years to the data set used in Filippini and Hunt (2012). For the purposes of this paper attention is restricted to the contiguous states (i.e. Alaska and Hawaii are excluded) except Rhode Island because of incomplete information: The District of Columbia is included and considered as a separate 'state'. The dataset is based on information taken from three sources. Residential energy consumption quantities and prices are provided by the Energy Information Administration (EIA). Population and real disposable personal income are from the Bureau of Economic Analysis of the US Census Bureau and the heating and cooling degree days are obtained from the National Climatic Data Center at NOAA. The number of housing units comes from the US Census Bureau and the share of detached houses for each state is based on the year 2000 census also obtained from the Census Bureau. Descriptive statistics of the key variables are presented in Table 4.

[Insert Table 4 here]

If we assume a Cobb-Douglas demand function, the econometric specification of the model can be written as:

$$\ln q_{it} = [\beta_0 + \beta_Y \ln Y_{it} + \beta_P \ln P_{it} + \beta_X \ln X_{it}] + g_{it} \cdot u_{it} + v_{it} \tag{10}$$

where subscript i stands for state, subscript t is time, $v_{it} \sim N^+[0,\sigma_v]$, and $u_{it} \sim N^+[0,\sigma_u]$. Our dependent variable (q_{it}) is each state's aggregate residential energy consumption for each year in trillion BTUs. The income variable (Y_{it}) is each state's real disposable personal income for each year in million 1982 US$. The price variable ($P_{it}$) is each state's real energy price for each year in 1982 US$ per million BTUs. The set of control variables X_{it} includes Population (POP_{it}), the heating and cooling degree days (HDD_{it} and CDD_{it}), the average size of a household (AHS_{it}) obtained by dividing population by the number of housing units, and the share of detached houses for each state (SDH_i).

Regarding the rebound effect function, this is modelled as a function of potential economic determinants of households' demand for energy services, such as household size, per capita income, and the price they must pay for energy. That is:

$$g_{it} = g_{it}(\gamma_0 + \gamma_Y \ln(Y/POP)_{it} + \gamma_P \ln P_{it} + \gamma_z \ln AHS_{it}) \tag{11}$$

If we impose that the rebound effect function does not depend on any covariate, we get the traditional and homoscedastic ALS model estimated in Filippini and Hunt (2011, 2012). Since the LOGISTC rebound effect function prevents unlikely rebound effect outcomes and attenuates the aforementioned identification issues, it is our preferred model. However the specification allowing for super-conservation outcomes, i.e. the RSCFG model, is also estimated for robustness purposes. All models are estimated by maximum likelihood.

We show in Table 5 the estimation results of our preferred frontier energy demand models. The traditional ALS model that imposes a zero rebound effects is also shown for comparison grounds. Simple Likelihood Ratio (LR) tests indicate that both the LOGISTIC and RSCFG models outperform the ALS model. In general, both models perform quite well as most coefficients have the expected sign and almost all are statistically significant at the 5% level. This indicates that the results in terms of the estimated coefficients tend to be robust across the two different specifications of the rebound effect. Although the performance of both models is very similar, a Vuong (1989) test that compares the goodness-of-fit of these two non-nested models indicates that the LOGISTIC model performs better than the RSCFG model at 5% of significance.

[Insert Table 5 here]

Regarding the energy demand frontier, the estimated coefficients can be directly interpreted as elasticities as most of the variables are in logarithmic form. The estimated magnitudes of both price and income elasticities are quite reasonable from a theoretical point of view. The estimated frontier coefficients suggest that US residential energy demand is price-inelastic, with estimated elasticities of -0.10, -0.12 and -0.11 for the ALS, LOGISTIC and RSCFG models respectively. The results also suggest that US residential energy demand is income-inelastic, with an estimated elasticity of around 0.36 for the ALS model but only about 0.24 for the models allowing for non-zero rebound effects.

The positive coefficient on population obtained in all models suggests that energy consumption increases with population, given the total amount of disposable income in a particular state. For weather, the estimated cooling degree day elasticities for all three models are rather high, whereas the estimated heating degree day elasticities are much lower. The estimated coefficient of average household size suggests that as family size increases there is a tendency to use less energy, indicating that there are economies of scale with an estimated elasticity larger than unity in absolute terms. For the share of detached houses, the results suggest that there is only a marginal positive but significant influence on US residential energy demand.[13]

Table 6 provides descriptive statistics of the estimated energy efficiency for all US states. The ALS values are obtained directly using the Jondrow *et al.* (1982) formula. For the LOGISTIC and RSCFG models, the efficiency scores are computed dividing the estimated value of the overall one-sided term (i.e. $g \cdot u$) by the estimated values of the rebound-effect function, g. We show three figures in Table 6 in accordance with the adjustments mentioned in Section 3. The first set of efficiency scores is obtained assuming that g has "no intercept" (i.e. $\gamma_0 = 0$) and hence u contains the whole

[13] As in Filippini and Hunt (2012), the estimated coefficients of the time dummies (not shown) are significant in all models and although the overall trend in the coefficients is generally negative, they do not fall continually over the estimation period, reflecting the 'non-linear' impact of technical progress and other exogenous variables.

estimated intercept. The second set of efficiency scores labelled "ALS-adjusted" follows the empirical strategy that uses the ALS estimate to adjust the intercept of g. The third efficiency scores are obtained following the opposite strategy to the first one, so in this case the rebound-effect function is "not adjusted". It is assumed that $\delta_0 = 0$ and hence u does not contain an intercept (i.e. σ_u is assumed to be equal to 1).

[Insert Table 6 here]

Table 6 shows that the estimated *average* efficiency is between 45.5% and 98.7%. However, this wide range of results is due to the models that consider that the intercept may either be in the rebound-effect function or in the inefficiency term. If we focus on the ALS-adjusted results, the values obtained with the LOGISTIC and the RSCFG models are much more reasonable (91.1% and 93.8% respectively). Similar results were obtained by Filippini and Hunt (2012) using several specifications of the homoscedastic model. It is worth mentioning that the ALS model produces similar efficiency scores to those obtained when the intercept is properly adjusted. The efficiency scores clearly decrease when the intercepts of the LOGISTIC and RSCFG rebound effect functions are not adjusted and the whole intercept is included in g. By contrast, the largest efficiency scores are obtained when no intercept is considered in the rebound effect function. These two cases define the lower and upper bound in the efficiency score estimates.

Regarding the rebound effect function, recall from Table 5 that the coefficients of both income per capita and price are always statistically significant. The theory on rebound effects often predicts that they should decline with income, and the coefficient of this variable is positive in both models. This implies that the states with larger income levels have larger energy efficiency elasticities in absolute values, and therefore their rebound effects are lower as they are obtained from R =1– g. This seems to confirm the aforementioned hypothesis and is in line with the little available evidence on this issue in the empirical literature measuring rebound effects. On the other hand, the negative coefficient obtained for the price variable suggests that energy-inefficient states have more elastic energy demands. This result is expected in theory as energy-inefficient states tend to spend a larger share of their income on energy *ceteris paribus*, and hence the so-called income effect is more intense.

Our comprehensive frontier model of energy demand allows us to examine the compliance with some of the restrictions often assumed in previous studies devoted to estimating rebounds effects, but with different econometric techniques. For instance, most studies estimate the own-price elasticity of the demand for energy to get an indirect measure of the rebound effect. Sorrell and Dimitropoulos (2008) pointed out that the estimated price elasticities might be biased if energy efficiency is not explicitly controlled for. The nature of this endogeneity problem is clear in our framework because the overall error term (which, if efficiency were ignored, would include energy efficiency) would be correlated with the energy price if the rebound-effect function relies on the energy price.

On the other hand, the validity of previous papers based on price elasticities hinge upon the assumption that consumers respond in the same way to decreases in energy prices as they do to improvements in energy efficiency. In particular, most of the empirical literature on rebound effects assumes that:

$$\varepsilon_E = -\varepsilon_P - 1 \tag{12}$$

We label this restriction as the *assumption of equivalence in responses*. Previous papers assume that equation (12) is fulfilled for all observations. As our model provides elasticities for both energy prices and energy efficiency, it allows us to examine (or even test) this issue in a very simple way. Thus, let us rewrite equation (12) as follows:

$$\varepsilon_P = a + b \cdot \varepsilon_E \quad , \quad a = b = -1 \tag{13}$$

Testing that $a = b = -1$ in an auxiliary regression allows us to examine the fulfilment of this assumption. In the Appendix we show that if we use a LOGISTIC specification of the rebound effect function, it is possible to directly test this assumption. In this sense, the Wald test carried out using the estimated parameters of our model suggests that energy and price elasticities are statistically different in our case. As a consequence, the absolute value of the elasticity of price in the frontier cannot be used for measuring the rebound effect as suggested by equation (12).

Table 7 provides descriptive statistics for the overall US estimated rebound effects using the LOGISTIC and RSCFG models. It should be recalled that there are no values larger than unity in the LOGISTIC model because its specification prevents backfire outcomes. In addition to the "ALS-adjusted" specification, for comparative purposes we show the estimated rebound effects that are obtained if the rebound-effect functions do not contain an intercept or if the estimated intercept is not adjusted (i.e. the estimated constant completely belongs to g). This table shows that the average rebound effect is 79% when our preferred LOGISTIC model is used and the intercept is adjusted using the standard deviation of u of the ALS model. It decreases to 56% when the RSCFG model is used.

[Insert Table 7 here]

Generally speaking, our rebound effects tend to be larger than those obtained in the empirical literature using micro-data on the direct rebound effects of household energy demand (see our discussion in Section 2). Two different issues can partially explain this result. First, note that our estimated rebound effects involve more than one energy service, and hence they are not only capturing direct but also indirect effects. In addition, it should be pointed out that our results are even lower than those obtained in several papers - such as Lenzen and Dey (2002), Alfredsson (2004) or Mizobuchi (2008) - that also get large direct and indirect rebound effects, even reaching effects larger than 100%, i.e. backfire outcomes. A second reason has to do with the curvature of the estimated rebound-effects functions. In Figure 1 it is shown that the proposed rebound-effect functions are concave, at least when the value of $\gamma' z$ in (9) tends to be negative, as happens in our case due to the negative value of the intercept and the fact that all variables have been centred with respect to the sample mean. Thus, our rebound effect estimates are likely to be upwardly biased because the curvature imposed on our g functions "forces" the rebound effect to increase rapidly when we move away from the zero value. Research devoted to finding more flexible yet still simple rebound-effect functions that relax this curvature would be desirable in the near future.

[Insert Figure 1 here]

On the other hand, it is worth mentioning that the estimated rebound effects in Table 7 is about 97% in both models when it is assumed that the estimated intercept completely belongs to g., Hence, contrary to what happens in the efficiency estimates this procedure gives an upper bound for the rebound effect. These extremely large values likely suggest that whereas g is downward biased, the estimated σ_u is upward biased (i.e. the true σ_u is likely less than 1). Assuming, by contrast, that the rebound effect function does not contain an intercept, the estimates produce an average rebound effect of about 50% for the LOGISTIC rebound effect function (and negative for the RSCFG model). This outcome is, however, due to the fact that all variables have been centred with respect to the sample mean, and thus we are imposing that g=0.5 for the *average* state. These results thus point out the importance of adjusting the estimated intercepts when computing rebound effects using an SFA approach.

Regarding the issue of allowing or not for super-conservation outcomes, Figure 2 shows the relationship between the ALS-adjusted rebound effects obtained using our proposed models. This figure reveals that the rebound effects in which super-conservation outcomes are not restricted (RSCFG model) are in practice monotonic transformations of the rebound effects obtained using models that only allow for partial rebound effects (LOGISTIC model). In other words, allowing for super-conservation outcomes only has an effect on the magnitude of the rebound effects, but not on the relative values across observations. Overall, these results indicate that the ranking of rebound effects tend to be robust across different specifications of the g function.

[Insert Figure 2 here]

In Table 8 we show the parameter estimates of both the LOGISTIC and RSCFG models when they are estimated without time dummies for robustness analysis. These models are presented to check the sensitivity of the approach proposed to measure rebound effects, as these dummies are likely to be capturing - among other common temporal effects - technological improvements in the energy efficiency of households' equipment and appliances over time.

[Insert Table 8 here]

Again, both models perform quite well as most coefficients have the expected sign and almost all of them are statistically significant. Secondly, the income per capita and price variables of the rebound effect function again have the expected signs and their coefficients are statistically significant. However, while the remaining coefficients are approximately in the same order of magnitude, the income and price elasticities vary notably. This result is particularly striking and highlights the importance of a proper specification of technical progress (using a time trend or temporal dummies) in order to obtain unbiased estimates of the price and income elasticities. This may be a significant problem especially in those analyses aiming at estimating rebound effects through the own price elasticity. Moreover, the coefficient of the price variable is negative and the coefficient of the income variable is positive, indicating that well-off states have lower rebound effects. In Table 9 we can see that both efficiency scores and rebound effects hardly change, indicating that the specification of technical progress in our model does not affect our results. As we have seen previously, the rebound effect function without adjustment and the rebound effect without an intercept show the lower and upper bounds respectively for both the efficiency score estimates and the rebound effect estimates. Encouragingly, these results indicate that, overall, the estimated efficiencies and rebound effects tend to be robust to the different specifications of the technical progress in the frontier.

[Insert Table 9 here]

Finally, our results might help policy makers to design more effective energy saving schemes. For instance, Figure 3 shows the overall relationship between energy efficiency and the rebound effect using our preferred model, the LOGISTIC specification. If we sort the US states according to their average efficiency scores and then check their average rebound effects, we can get an idea about the correlation between these two measures. The average energy efficiency of the states in the fourth quartile is 86.3%. As usual in a frontier analysis framework, energy savings are *potentially* larger in those states with lower efficiency scores. Unlike standard SFA models, our models allow us to know whether the potential reductions in energy inefficiency are passed on entirely to final energy savings. As the states of the fourth quartile have also the lowest rebound effect (56.7%) we have more reasons to encourage energy efficiency improvements in these states. On the other hand, it is worth mentioning that although efficiency and rebound effects tend to increase as we move down the quartiles, the gap between both measures decreases and reaches a minimum difference in the first quartile where the most energy-efficient states (93.3%) are also those with the largest rebound effect (90.9%). This result indicates that as the efficiency of US states increases, households are less sensitive to changes in efficiency and they do not reduce their energy consumption as much as would be expected if we are swayed by what happens to the states with lower levels of efficiency.

[Insert Figure 3 here]

Focusing on the minimum rebound effects on Table 7, we can see that although the rebound effect is large on average, some US states have very small rebound effects compared to others. It can be seen in Figure 3 that there is a clear correlation between energy efficiency and rebound effects, but this does not mean that large energy efficiencies necessarily imply large rebound effects. Figure 4 reveals the heterogeneity that exists in our US sample. Those states with low energy efficiency (below the median) and a low rebound effect (also below the median) are highlighted in dark orange. These states are identified here as priority targets for energy policies, since improvements of energy efficiency in these states may yield large reductions in energy consumption (and probably greenhouse gas emissions). [14] On the other hand, those states marked in the lightest orange have large energy efficiencies as well as large rebound effects and therefore they should be labelled as the lower-priority targets. The intermediate orange highlights those states that have either low energy efficiency or a low rebound effect and hence cannot be identified as priority objectives. In summary, a sound policy would be not only focused on the most inefficient states but also on those with low rebound effects where the policy would have a greater overall effect over energy consumption.

[Insert Figure 4 here]

5. Conclusions

This paper highlights that the energy demand frontier model, originally proposed by Filippini and Hunt (2011, 2012) for obtaining energy efficiency scores of a sample of countries, is closely linked with the measurement of the so-called rebound effect. This phenomenon associated with energy consumption captures the idea that an increase in the energy efficiency level of a certain energy service is perceived by the consumers as a lower price and hence may lead to an increase in the consumption of that service.

As stressed in this paper, the standard (homoscedastic) specification of the energy demand frontier model basically imposes a rebound effect equal to zero,

[14] It should be stressed that if the average value is used instead the median to classify the states, just seven (Connecticut, Illinois, Maryland, Massachusetts, New Jersey, New York and Utah) would be below the average value of both efficiency and rebound effect, and hence only these would be primary targets.

something that clashes with the empirical evidence obtained in the literature on the rebound effect. We point out in this paper that we can obtain a direct measure of the rebound effect and relax the 'zero' rebound assumption through a convenient reinterpretation of a (heteroscedastic) stochastic frontier model that satisfies the so-called scaling property, which is well-known in the literature on productivity and efficiency analysis. We have also pointed out that our reinterpretation of a stochastic frontier model opens new identification issues that do not appear in the standard stochastic production frontier literature. The nature of these issues has been discussed, and a simple solution is proposed based on ALS model estimates.

The fulfilment of the scaling property implies in our framework that the traditional one-sided random term can be decomposed into a *pure* random term capturing the underlying level of energy inefficiency which in turn is scaled upwards or downwards by a deterministic function that directly measures rebound effects. Two specifications for the rebound-effect function that preclude backfire outcomes are presented in the paper. While the RSCFG model allows for somewhat counterintuitive super-conservations outcomes, the LOGISTIC model only allows for partial rebound effects.

In contrast to the previous literature that treat the above deterministic functions as energy inefficiency, we point out in this paper that they could be (also) capturing a rather different in nature phenomenon, i.e. the rebound effects associated to improvements in energy inefficiency. We call attention to the fact that the traditional energy efficiency scores should be interpreted with caution under the presence of rebound effect phenomena, and that the design of effective energy-saving policy measures should take into account the different nature of both issues.

We illustrate the approach proposed to measuring rebound effects with an empirical application of US residential energy demand data for 48 states over the period 1995-2011. The coefficients of the variables included in the models are highly significant, show the expected signs and have a quite reasonable magnitude regardless of the specification of the rebound effect function used. Regarding the efficiency scores there is not much variation between estimated (LOGISTIC and RSCFG) models and they do not change much in response to the different strategies used to obtain the intercept of the rebound effect function.

In relation to the rebound effects, values that are too large and too low are obtained if we ignore the identification issues discussed, i.e. if we assume that the estimated intercept completely belongs to the rebound-effect function, or that this function does not contain any intercept. Regarding the functional form of the rebound effect, we advocate using the constrained functional form- the LOGISTIC model - because it is not only preferred to the RSCFG model on statistical grounds but also avoids the issue of obtaining super-conservation results for some observations that are too large. Although the estimated rebound effects vary with the functional form, the position of each observation does not change as the RSCFG rebound effects is a monotonic transformation of the rebound effects obtained with the LOGISTIC model. This is an important result as the relative position of each state in terms of both energy efficiency and rebound effect rankings permits the identification of states where the enforcement of policies with the aim of promoting energy efficiency would be more effective. Unlike those analyses aiming at estimating rebound effects through the own price elasticity, another interesting feature of our empirical approach is that it suffers less from biases when technical progress is ignored.

Further work remains to be done regarding the estimation of rebound effects using a stochastic frontier approach. A key issue is the identification of the true intercept of the rebound-effect function. While we have proposed a simple empirical strategy to split the estimated intercept into its two components, two alternative approaches could be used: one aims to select the observations where the homoscedastic (or ALS) model cannot be rejected, whereas the other procedure relies on assuming that the rebound-effect function is a positive random variable similar to the inefficiency term. While the first approach can be implemented using a latent class model, the second approach would imply the estimation of a stochastic frontier model with two multiplicative one-sided errors terms. Another issue has to do with the concavity problems of the proposed rebound-effect functions, which tend to overestimate the rebound effect. Although this is likely an issue related to our data set, future research will be likely focused on the use of alternative parametric specifications of the rebound function that prevent upward biased rebound effects. In this sense, it should be also explored the potential use of semiparametric regression methods to relax the current concavity constraints.

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Table 1. Approaches for measuring the direct rebound effect

Approach 1	$\varepsilon_E(q) = \varepsilon_E(S) - 1$
Approach 2	$\varepsilon_E(q) = -\varepsilon_{P_S}(S) - 1$
Approach 3	$\varepsilon_E(q) = -\varepsilon_{P_q}(q) - 1$

Notes: Letters in parentheses stand for elasticity numerators and subscripts for elasticity denominators
S: Useful work; q: Energy; P_S: Energy cost of useful work; P_q: Energy price; E: energy efficiency

Table 2. Possible values for the rebound effect and the energy efficiency elasticity

R > 1	Backfire	$\varepsilon_E > 0$
R = 1	Full rebound	$\varepsilon_E = 0$
0 < R < 1	Partial rebound	$-1 < \varepsilon_E < 0$
R = 0	Zero rebound	$\varepsilon_E = -1$
R < 0	Super-conservation	$\varepsilon_E < -1$

Table 3. Proposed rebound effect functions

Super-conservation	***Specification***
No	LOGISTIC $g = \dfrac{exp(z'\gamma)}{1 + exp(z'\gamma)}$
Yes	RSCFG $g = exp(z'\gamma)$

Table 4. Summary statistics of variables

Variable	***Description***	***Mean***	***Std. Dev.***	***Min.***	***Max.***
Q	Energy Consumption	229.60	209.42	19.02	932.92
Y	Income	92,620	105,635	6,072	654,780
P	Energy price	16.86	5.11	8.22	35.18
POP	Population	5,977	6,407	485	37,692
HDD	Heating degree days	5,134	2,007	555	10,745
CDD	Cooling degree days	1,147	805	128	3,870
AHS	Household size	2.33	0.17	1.83	2.99
SDH	Share of detached houses	62.3	9.7	13.2	74.0

Table 5. Parameter estimates (models with time dummy variables)

	Parameters	ALS			LOGISTIC			RSCFG		
		Est.		***s.e.***	***Est.***		***s.e.***	***Est.***		***s.e.***
Frontier										
	Intercept	5.012	***	0.022	5.043	***	0.018	5.042	***	0.018
	ln (Y)	0.364	***	0.037	0.238	***	0.046	0.236	***	0.046
	ln (P)	-0.101	***	0.025	-0.117	***	0.030	-0.114	***	0.030
	ln (POP)	0.670	***	0.038	0.797	***	0.047	0.799	***	0.047
	ln (AHS)	-1.117	***	0.053	-1.480	***	0.086	-1.469	***	0.088
	ln (HDD)	0.373	***	0.013	0.347	***	0.013	0.348	***	0.013
	ln (CDD)	0.084	***	0.007	0.080	***	0.008	0.080	***	0.008
	SDH	0.005	***	0.001	0.005	***	0.001	0.005	***	0.001
Noise term										
	ln (σ_v)	-2.633	***	0.120	-2.554	***	0.036	-2.555	***	0.037
Inefficiency term (Heteroscedastic)										
	Intercept				-4.281	***	0.714	-4.124	***	0.670
	ln (Y/POP)				7.014	***	2.242	6.148	***	2.034
	ln (P)				-1.577	*	0.862	-1.446	*	0.769
	ln (AHS)				14.283	***	3.640	12.187	***	3.165
Inefficiency term (Homoscedastic)										
	ln (σ_u)	-2.530	***	0.258						
Log-likelihood		842.185			875.919			874.948		

Notes: ***, ** and * indicate that the coefficient are significant at 1%, 5% and 10%.

Table 6. Energy efficiency scores using the LOGISTIC and RSCFG models
(models with time dummy variables)

	Mean	*Std. Dev.*	*Min.*	*Max.*
ALS				
	0.939	0.025	0.831	0.977
LOGISTIC				
No intercept (γ_0 = 0)	0.964	0.040	0.703	0.989
ALS-adjusted (γ_0 = -1.833)	0.911	0.041	0.650	0.958
Not adjusted (γ_0=-4.281)	0.455	0.078	0.202	0.847
RSCFG				
No intercept (γ_0 = 0)	0.987	0.003	0.974	0.997
ALS-adjusted (γ_0 = -1.593)	0.938	0.013	0.880	0.987
Not adjusted (γ_0=-4.124)	0.455	0.079	0.200	0.848

Table 7. Rebound effects using the LOGISTIC and RSCFG models
(models with time dummy variables)

	Mean	*Std. Dev*	*Min.*	*Max.*
LOGISTIC				
No intercept (γ_0 = 0)	0.505	0.269	0.033	0.982
ALS-adjusted (γ_0 = -1.833)	0.791	0.208	0.177	0.997
Not adjusted (γ_0=-4.281)	0.966	0.052	0.713	1.000
RSCFG				
No intercept (γ_0 = 0)	-1.178	-2.131	-17.866	0.969
ALS-adjusted (γ_0 = -1.833)	0.557	0.637	-2.835	0.994
Not adjusted (γ_0=-4.124)	0.965	0.949	0.695	1.000

Table 8. Parameter estimates (models without time dummy variables)

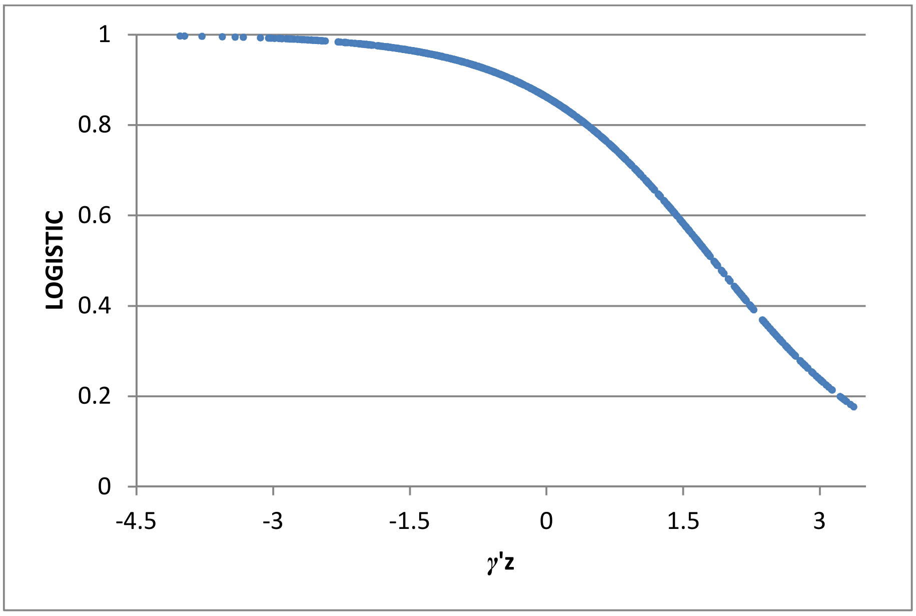
	Parameters	ALS			LOGISTIC			RSCFG		
		Est.		***s.e.***	***Est.***		***s.e.***	***Est.***		***s.e.***
Frontier										
	Intercept	4.937	***	541.662	4.992	***	646.179	4.990	***	597.731
	ln (Y)	0.259	***	7.759	0.114	***	2.750	0.113	***	2.716
	ln (P)	-0.207	***	-12.496	-0.198	***	-9.497	-0.196	***	-9.402
	ln (POP)	0.776	***	22.100	0.921	***	21.421	0.923	***	21.483
	ln (AHS)	-1.113	***	-19.282	-1.430	***	-17.854	-1.422	***	-17.517
	ln (HDD)	0.353	***	27.950	0.326	***	26.830	0.326	***	26.829
	ln (CDD)	0.079	***	11.127	0.070	***	9.485	0.069	***	9.390
	SDH	0.004	***	7.707	0.004	***	7.912	0.004	***	7.805
Noise term										
	ln (σ_v)	-2.738	***	-25.362	-2.518	***	-70.482	-2.520	***	-69.124
Inefficiency term (Heteroscedastic)										
	Intercept				-4.014	***	-6.857	-3.881	***	-7.205
	ln (Y/POP)				6.855	***	3.552	5.979	***	3.476
	ln (P)				-1.326	*	-1.785	-1.190	*	-1.829
	ln (AHS)				12.592	***	4.061	10.719	***	3.976
Inefficiency term (Homoscedastic)										
	ln (σ_u)	-2.239	***	-19.221						
Log-likelihood		804.455			839.950			839.191		

Notes: ***, ** and * indicate that the coefficient are significant at 1%, 5% and 10%.

Table 9. Energy efficiency scores and rebound effects using the LOGISTIC model (model without time dummy variables)

	Mean	***Std. Dev***	***Min.***	***Max.***
Energy Efficiency Scores				
No intercept	0.957	0.044	0.671	0.985
ALS-adjusted	0.886	0.046	0.603	0.955
Not adjusted	0.456	0.083	0.150	0.860
Rebound effects				
No intercept	0.506	0.258	0.042	0.970
ALS-adjusted	0.805	0.190	0.224	0.995
Not adjusted	0.961	0.054	0.708	0.999

Figure 1. Curvature of the estimated rebound-effect functions



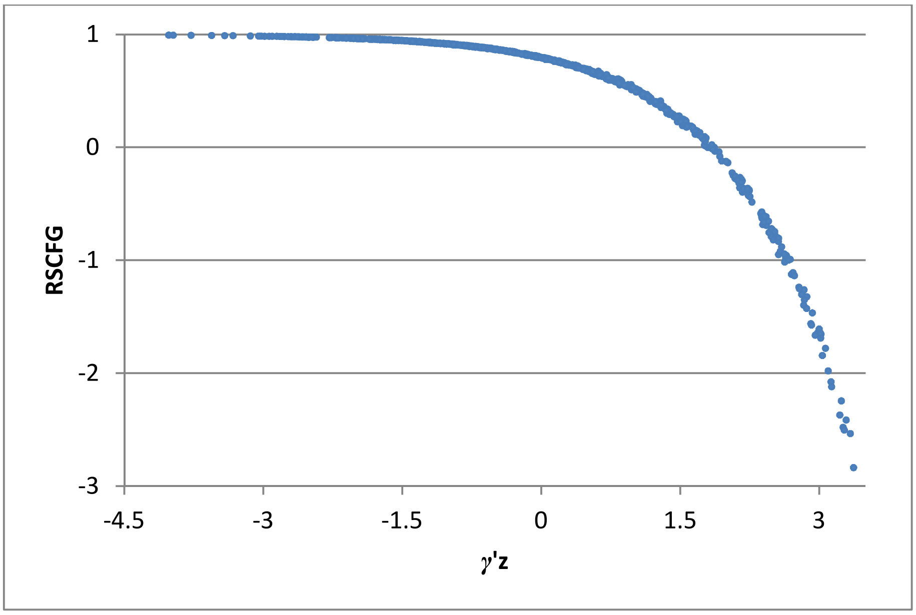


Figure 2.
Rebound effects with and without super-conservation outcomes (ALS-adjusted intercepts)

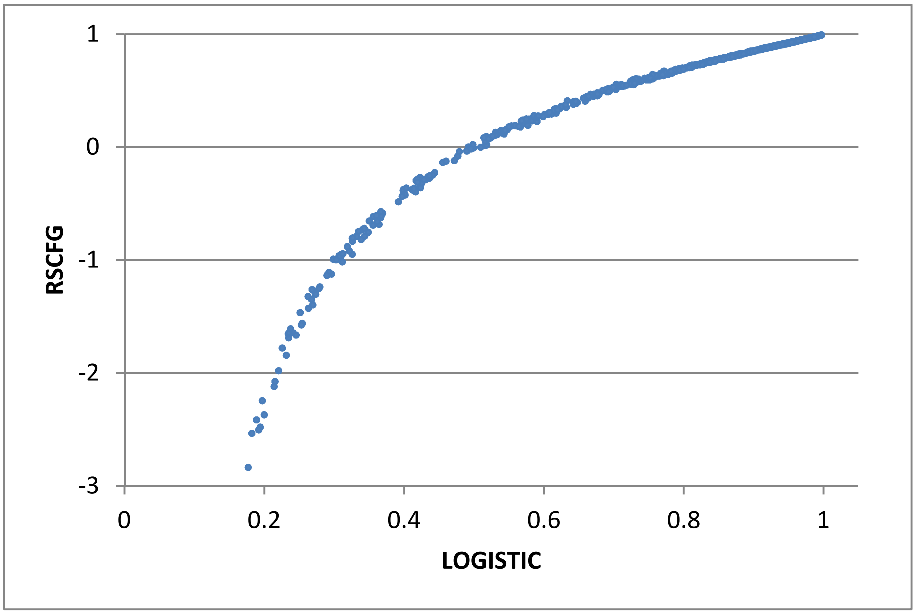


Figure 3. Average energy efficiency scores and rebound effects using the LOGISTIC model

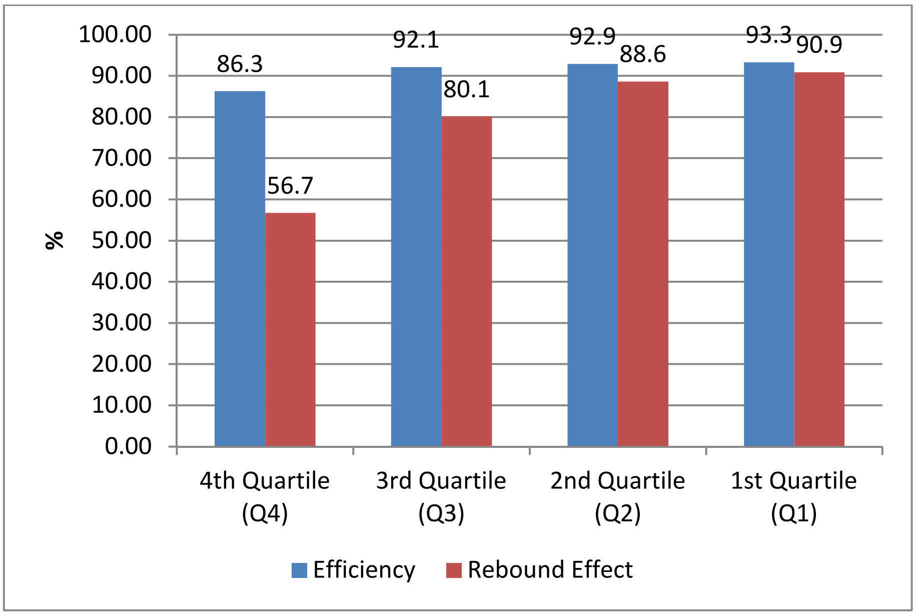
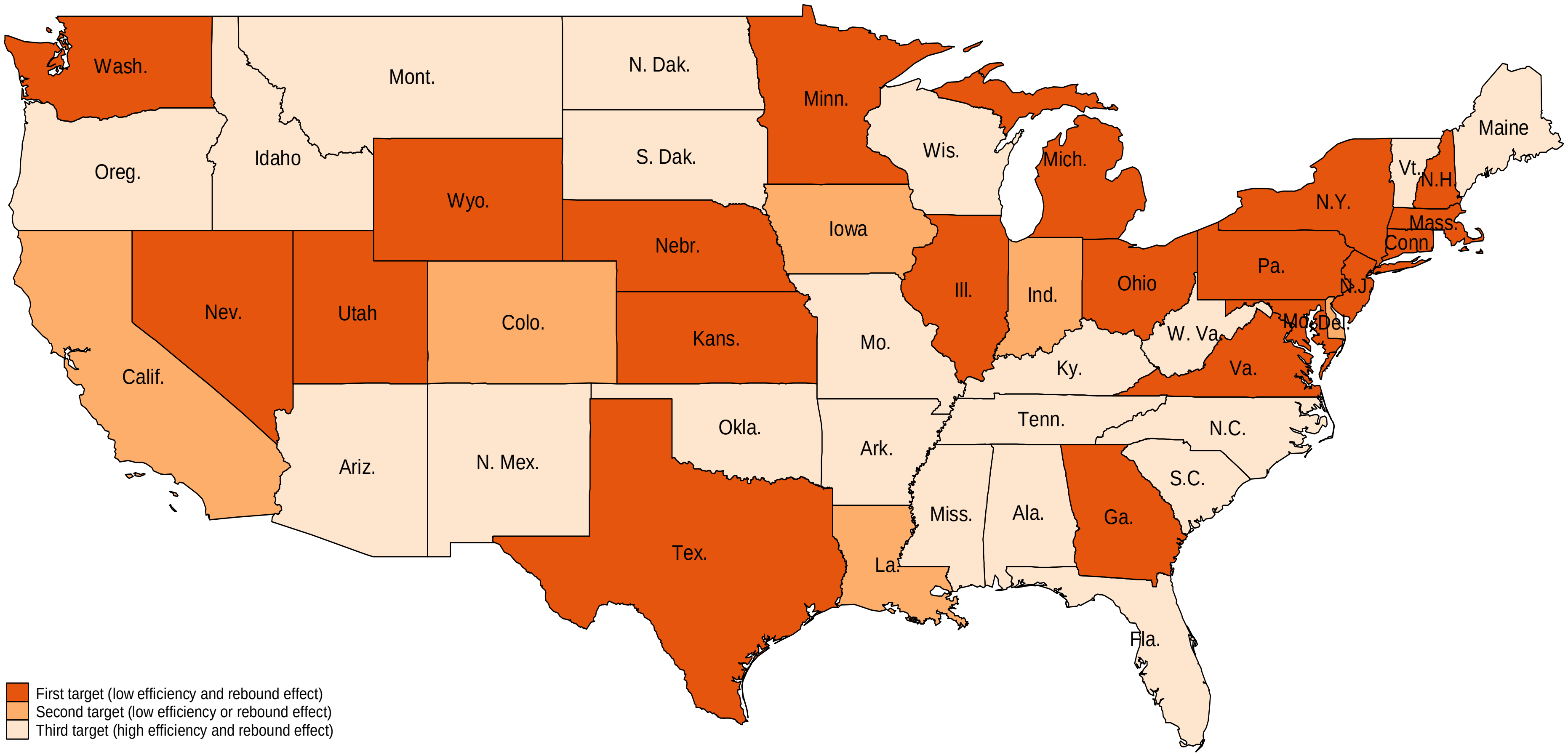


Figure 4. Map of US states in which priority targets to reduce energy consumption are identified



APPENDIX

Testing the assumption of equivalence in responses using a logistic rebound effect function

Let us assume that the demand function is Cobb-Douglas and that the rebound effect function is exponential. In this case, the price elasticity of energy demand can be written as:

$$\varepsilon_P = \beta_P - \gamma_P \cdot g(Y, P, \gamma) \cdot \ln E \tag{A1}$$

where β_P is the *frontier* price elasticity and γ_P is the coefficient of ln P in the exponential rebound effect function. As $\varepsilon_E = -g$, equation (A1) can be rewritten now as follows:

$$\varepsilon_P = \beta_P + (\gamma_P \cdot \ln E) \cdot \varepsilon_E \tag{A2}$$

In summary, equations (10) and (A2) jointly indicate that the equivalence of responses assumption will be satisfied in our model if we cannot reject the following null hypothesis:

$$H_0\text{: } \hat{\beta}_P = \hat{\gamma}_P \cdot \ln E = -1 \tag{A3}$$

Testing this hypothesis is difficult as energy efficiency varies across states and over time. An alternative way to test the equivalence of responses assumption is to test a sufficient (but weaker) condition for the fulfilment of the above hypothesis evaluated at the estimated mean of the energy inefficiency term:

$$H_0\text{: } \hat{\beta}_P + \hat{\gamma}_P \cdot \hat{E}(u) = 0 \tag{A4}$$

As we assume that u=-ln$E$ follows a half-normal distribution, the expected mean in (A4) is simply a function of σ_u. and hence the sufficient condition in (A4) can be finally expressed as follows:

$$H_0\text{: } \hat{\beta}_P + \hat{\gamma}_P \cdot \sqrt{2/\pi} \cdot \hat{\sigma}_u = 0 \tag{A5}$$

If instead we use a logistic rebound effect function, the sufficient condition in (A4) becomes:

$$H_0\text{: } \hat{\beta}_P + \frac{\hat{\gamma}_P}{1+exp(z\prime\hat{\gamma})} \cdot \sqrt{2/\pi} \cdot \hat{\sigma}_u = 0 \tag{A6}$$