The Turning Tide: How Energy has Driven the Transformation of the British Economy Since the Industrial Revolution

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Abstract

Since the Industrial Revolution, the economy of the UK has transformed from that of an industrial manufacturing giant to a service economy and a central hub for the financial sector. Energy and energy services derived from fossil fuels have played a key role as drivers behind this structural change. Using data from 1855—2015 on capital, labor, and energy in a CES production function, we show that during this period input factors were mostly gross complements. However, between 1960 and 1980, the elasticity of substitution of energy increased substantially, from around 0.7 to more than 2.4. These high elasticity estimates were not permanent, and this wave of change that characterized the transition has since dissipated. Elasticities have since returned to even lower values around 0.3, indicating that energy services which depend primarily on fossil fuel inputs, such as transportation, pose a serious limit to the efficacy of efforts aimed at reducing fossil fuel consumption.

JEL classification: C55, E13, E23, N10, Q43

Keywords: Elasticity of substitution; Energy inputs; Aggregate Production; Industrialization; Structural change;
1 Introduction

In the last 160 years, the United Kingdom has undergone a remarkable transformation, morphing from the country that was the birthplace of industrialization to one of the first western industrialized countries that actively strove to change itself into a service economy. At the same time, it turned from one of the largest coal producers to a country that just recently had its first day where coal was no longer used for electricity generation. This transition provides researchers with the opportunity to investigate and better understand how these changes manifest in the parameters of a production function. The changes in the elasticity of substitution are of particular interest here, since increases in the elasticity between factors can be a driver for growth. This holds especially true for the role of energy. There is a strong hope among policy makers around the world that developed economies will be able to ween themselves off fossil fuel consumption for good without sacrificing economic performance, by investing in an energy-efficient capital stock that is flexible and effective at utilizing renewable energy sources.

Using 161 years of macroeconomic data, dating from 1855 to 2015, on labor use and wages, capital stock and capital costs, as well as fossil fuel consumption and expenditure data for coal, natural gas, and oil in the UK, we analyze the development of technical change and the elasticity of substitution between input factors. We utilize a nested CES production function with the inputs of labor, capital, and energy, and estimate it by using the system of equations approach of León-Ledesma et al. (2010), which was adapted for use with three input factors in Frieling and Madlener (2016). We test the assumptions of stability that define a CES function by comparing the technical change and elasticity estimates when using the whole sample from 1855—2015, the two halves of the sample from 1855—1935 and from 1935—2015, as well as the results of a sliding 51-year window starting with 1855—1905. We find that the parameter estimates were remarkably stable for the majority of the sample. However, the transformations forced upon the UK by the back-to-back energy shocks of a rapid decline of the coal industry and two oil crises swept like a wave through the economy. During this transitional period we observe a radical increase in the elasticity of substitution between energy and the other production factors as a result of the reshaping of the economy. In the wake of these events, however, elasticities are even lower than they were before.

Our research ties into existing work on energy and growth, such as Kander and Stern (2014), who analyzed the use of different energy carriers and their substitution during the period of industrialization in Sweden, and found that energy and other inputs are gross complements. Our research adds to this by providing empirical evidence that the elasticity of substitution can increase dramatically.
as a result of structural change. In light of studies such as Klump and de la Grandville (2000) and Klump et al. (2011), where it is posited that increases in elasticity can boost growth, the fact that large changes prove to be only transient is important. It also adds some more economic context to the studies by Fouquet (2011) and Fouquet (2014). Those studies highlight the persistent demand for “outdated” energy carriers even as more efficient and cheaper energy sources became widespread.

Our empirical results indicate that energy-specific technical change happened in two spurts, the first of which coincides with the rapid expansion of the availability of fossil fuels for the whole economy at the beginning of the 20th century. It reflects the incorporation of new technologies in the wake of widespread electrification, and the replacement of capital stock designed according to the necessities of the steam engine (David, 1990). The second spurt of development came with the shift away from a coal- and industry-reliant economy towards an economy relying much more on the service sector, in particular the financial services sector. We theorize that reduced elasticity of substitution of energy, which we find in the aftermath of the 1970s, is the result of a persistent demand for energy services which still rely heavily on fossil fuels, such as transportation. These services are dependent on certain energy inputs, and even if these inputs become expensive, technological barriers prevent their effective substitution.

The paper extends the growing literature on the empirics of normalized production functions. The results show that over very long periods, shifts in the underlying structure of economic activity can mean that the performance of the CES framework deteriorates. We find that substitution elasticities tend towards unity in the very long run, as the exogenous pressures on the varying factors lead to endogenous changes in the focus of technical change and substitution possibilities. In order to have an indication as to how unique the results for the UK are, we compare the second half of the sample to the case of the US, examined in Frieling and Madlener (2017), which covers a similar time frame. The analysis provides clues as to the reasons for the different developments in factor shares and technical change between the countries. We find that, overall, the energy mix in the US remained much more stable than in the UK, while a smaller reliance on foreign energy imports reduced the effects of the oil crises of the 1970s and 1980s compared to the UK.

In the following, we first describe the data set and highlight historical developments in Section 2. Section 3 describes the CES production function and the system of equations used for the estimation. The results for the long-run estimations and a comparison with the US can be found in Section 4, and these are contrasted by the moving averages in Section 5. Section 6 concludes.
2 Data

Due to the institutional continuity and territorial stability, researchers have access to very rich and consistent datasets for the UK, some of which cover multiple centuries. The data used in this study spans the 161 years from 1855—2015, and includes the disruptions caused by two world wars, rapid industrialization, the availability of new and widespread energy sources, major strikes, and the independence of the Republic of Ireland in 1920. We opted to include all the data and do not correct for breaks in the series, as it is unclear whether all input factors are equally affected by breaks, and we would otherwise lose valuable information from the adjustment processes following such a break. We rely on the data published in the national accounts, called the “Blue Book”, as well as historical datasets, mainly those in Mitchell (1988). We also use data from the project “Three centuries of British data”, compiled by the Bank of England (BoE), which is an aggregation of a number of historical datasets that are spliced to create comparable long-term time series. Our analysis is mainly constrained by the availability of reliable energy consumption and price data. For data on energy use and prices we rely on datasets from Fouquet (2011) and Fouquet (2014), as well as the Digest of UK Energy Statistics (DUKES).

The output data is measured in millions of £ and converted to real terms using prices from 2000. We define gross output as GDP plus energy expenditures, following the method of Kander and Stern (2014). We are able to subtract from GDP capital income from dwellings, which allows us to better analyze the development of the productive capital stock. This is especially important in the UK, where the lack of major destruction through wars and rapid urban development in the 19th century have created a surprisingly stable housing stock providing a steady stream of rents. We assume that output is used according to the accounting identity

\[ Y \equiv wL + rK + pE, \]  

where \( Y \) is gross output and \( L, K, E \) denote the factor inputs labor, capital, and energy. The factor inputs are paid the factor costs of \( w, r, p \), which represent the labor compensation, the capital returns, and the aggregate energy price.

The capital stock series counts the net capital stock excluding dwellings and including depreciation and is evaluated at repurchasing cost using prices from 2000. The massive changes in the composition

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1In the UK, 75% of dwellings are older than 35 years and almost 21% were built before 1919. In the US, only 6% of the housing stock is from before 1919, while 58% is younger than 50 years. In Germany, despite the widespread destruction of WWII, 23% of the occupied housing stock is from before 1919 and 74% of housing stock is older than 40 years.
of the capital stock over such a long period of time make this conversion difficult, and we rely on the spliced estimate compiled by the BoE and the data from Mitchell (1988), as well as Oulton and Wallis (2016). The capital share of factor incomes is what accrues after energy expenditure and wages are accounted for. Due to the difficulty of directly estimating the capital return series \( r \), we derive capital income from the accounting identity (1). A comparison with other available studies on capital returns, such as Piketty and Zucman (2014) and Oulton and Wallis (2016), shows that this method is consistent with their results.

The data for labor use includes employees and self-employed persons. Labor compensation in the study is derived from the total labor compensation reported in the Blue Books, and compiled in the three centuries dataset, but is scaled up to account for the labor price for self-employed persons. This is necessary because the income of self-employed persons consists of labor and capital income. The labor compensation of employees can therefore be used as a shadow-price of labor to approximate the hypothetical wages a self-employed person would receive (León-Ledesma et al., 2015).

Data on energy use is the constraining factor for this analysis, as the nature of energy use has shifted dramatically since 1855. We limit ourselves to analyzing the three main fossil fuel sources of coal, natural gas, and oil, because there is consistent and comprehensive data on prices and consumption. This excludes a number of energy sources that have played a role for production, such as animal and human muscle power, renewable power in the form of wind energy, solar power, or hydroelectric power. However, by 1855 the UK was already a heavily industrialized economy. At this time, 95.5% of primary energy consumption was coming from coal alone (Fouquet, 2014). The fossil fuel dependence peaked in 1955, when 99.2% of energy consumption was derived from fossil fuel sources. The overall share in fossil fuel dependence remained relatively stable until the 1970s, when it began to slowly decline. According to DUKES data, fossil fuel dependence shrank from 96.5 % in 1970 to just 82.0% in 2015, with nuclear and renewable energy accounting for this reduction. We consequently account for the overwhelming majority of all energy consumption during the sample period.

We convert all consumption data to tons of oil equivalent (toe), in order to ensure the comparability across time and to generate a consistent aggregate measure in the form of total toe consumption. We use data on coal consumption from Mitchell (1988), who reports local production, and from which we subtract net exports and coal used for international shipping. Coal price in £/toe is derived from multiple sources, for the years 1855—1935 from expenditure, corroborated by data from Fouquet (2011) and Fouquet (2014), who also relies on Mitchell (1988). For the years 1935—1970 the series for coal prices at the pithead reported by Fouquet (2011) is used, while the DUKES price index is
used for prices from 1970—2015.

Gas consumption changed considerably over the time period considered. In the 1800s, almost all gas came from manufactured gas, also called “town gas”. It was mostly used for lighting, first for street lights, and later on in private homes. It was usually produced locally from coal and sometimes from wood in so-called gas works. Because this type of gas is derived from other energy carriers (mostly coal, but also wood in some instances), it is not counted separately in our analysis. Only in the second half of the 20th century did the consumption of natural gas become important, particularly after discoveries of deposits off the coast of Scotland. The price series from Fouquet (2014) is spliced from town gas and natural gas prices; however, since natural gas consumption was effectively zero at the beginning of the sample until the 1920s, this does not factor into the overall consumption and expenditure data. The data on oil prices and consumption are also taken from Fouquet (2014). The price data has been corroborated from the BP World Energy Outlook and the expenditure data reported in the DUKES tables. Prices for oil, gas, and coal are not further disaggregated along different qualities.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>Y</td>
<td>Output as real GDP + energy expenditure in million £, 2000 prices.</td>
</tr>
<tr>
<td>Labor</td>
<td>L</td>
<td>Labor as employees + self-employed, in 1000.</td>
</tr>
<tr>
<td>Capital</td>
<td>K</td>
<td>Net capital stock in billion £, 2000 prices.</td>
</tr>
<tr>
<td>Energy</td>
<td>E</td>
<td>Sum of energy consumed per year in Mtoe.</td>
</tr>
<tr>
<td>Coal</td>
<td></td>
<td>Annual coal consumption, converted from short tonnes to Mtoe.</td>
</tr>
<tr>
<td>Oil</td>
<td></td>
<td>Annual crude oil consumption, converted from barrels to Mtoe.</td>
</tr>
<tr>
<td>Nat. gas</td>
<td></td>
<td>Annual natural gas consumption, converted from cubic feet to Mtoe.</td>
</tr>
<tr>
<td>Wages</td>
<td>w</td>
<td>Total employee compensation in real £, 2009 prices.</td>
</tr>
<tr>
<td>Capital cost</td>
<td>r</td>
<td>Real user cost of capital derived from the accounting identity (1).</td>
</tr>
<tr>
<td>Energy price</td>
<td>p</td>
<td>Average real price (in £/toe).</td>
</tr>
<tr>
<td>Coal price</td>
<td></td>
<td>Real coal price (in £/toe).</td>
</tr>
<tr>
<td>Oil price</td>
<td></td>
<td>Real oil price (in £/toe).</td>
</tr>
<tr>
<td>Nat. gas price</td>
<td></td>
<td>Real gas price (in £/toe).</td>
</tr>
<tr>
<td>Quality</td>
<td>Q</td>
<td>An index indicating the overall quality of the fuel mix.</td>
</tr>
</tbody>
</table>

2.1 Energy developments

The dataset covers a period of rapid changes and developments in the British economy. This holds particularly true for the energy environment. Fouquet (2011) highlights the diverging prices of energy services and energy sources, and the distinction is important to understand the within-factor substitutions that take place under the surface. Energy services have different requirements placed
upon them than just their price and efficiency. Provender, meaning horse feed, was a key agricultural energy expenditure well into the beginning of the 20th century and accounted for 60% of the agricultural expenditure on energy for power in 1900 (Fouquet, 2011). The reason is that the energy service required for farming is that of the compact and mobile delivery of mechanical power. Efficiency gains for steam engines and the huge improvements in turbine technology for generating electricity, or in the efficient distribution of town gas through gas mains, did very little to improve energy productivity in the agricultural sector. Only with the advent of mechanized farm equipment utilizing compact internal combustion engines did fossil fuels significantly change the agricultural sector. Going forward, this tendency to think in services will be important, because moving away from emission-intensive internal combustion engines for transportation is difficult if the alternatives do not qualify for the demands placed upon the service they are supposed to replace. For example, electric mobility’s ability to replace internal combustion engines depends on whether the demands on the service can be met, rather than just on the efficiency of the vehicles alone.

Unfortunately, data on energy services is only available for very isolated cases (see Fouquet (2011, 2014)). Additionally, there are usually no observable prices for energy services. This means that for production functions it is usually necessary to measure fuel inputs instead. One technique to improve the measurement of effective energy inputs was proposed by Stern and Kander (2012) and Kander and Stern (2014). They construct a quality index which accounts for the fact that the heat units of different energy carriers are not actually equivalent from an economic perspective. As is evident from the farmers who still paid a premium for provender, not all energy services are available for use with all fuels. The prices of the fuels, therefore, at least partially reflect the economic value of the services that can be produced with those fuels. The quality index is calculated as the ratio of a Fisher chain quantity index of the different energy sources with a simple index of the heat units. This means that the quality index increases even when the consumption in terms of heat units stays constant, as long as there is a substitution within the energy inputs towards more expensive fuels. While the quality index is only a rough approximation of the development of energy services, in light of the dramatic shifts in the energy mix it seems appropriate to include it in the analysis. The developments of the quality index show that there are significant shifts in the types of fuel sources that are used in the British economy. As the consumption of oil and gas increases, the overall quality of the fuel mix increases as well.

Especially at the beginning and end of the sample there is a portion of energy consumption which

\[^2\text{For a more in-depth treatment of the quality index, see Kander and Stern (2014) and Hulten (1973).}\]
we do not consider. This is the renewable energy made up of animal and human muscle power and
fueled by food and feed, hydro power from watermills, wind power used by windmills and enabling
transportation in the form of sailing, as well as energy from firewood used for heating and lighting.
However, even when just accounting for fossil fuels, the British energy demand expanded significantly
in the first half of the sample. In 60 of the 80 years between 1855 and 1935, energy demand measured
in Mtoe grew more than the GDP. If we consider the whole sample, the British economy grew by
2.0% annually, whereas fossil fuel consumption in Mtoe only grew by .96% p.a.

The energy share, as measured by expenditures on fossil fuels, has fluctuated substantially in the
series, starting low at just 4.2%, but reaching levels as high as 14.9% in 1984, meaning that almost a
sixth of the overall value added went to the consumption of fossil fuels. The individual energy sources
also underwent substantive changes, as is evident in Table 2. Gas prices were highest at the start of
the sample, when the process of manufacturing gas from coal was first invented and implemented in
large urban centers. Similarly, oil only began to play a large role in the 20th century, when the wide
spread of internal combustion engines led to a large increase in fuel demands.

Interestingly, the British economy seemed to have reacted to the oil crisis in a much more radical
way than the US economy. The energy demand reached a maximum of 212.7 Mtoe in 1973, and
rising energy prices depressed the capital share to just above 20% in 1975. This sparked the massive
restructuring of the British economy under the government of Prime Minister Margaret Thatcher,
leading to a deindustrialization and a shifting of the focus onto the service sector.

| Table 2 |

<table>
<thead>
<tr>
<th>Descriptive energy statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Value</strong></td>
</tr>
<tr>
<td>Quality index</td>
</tr>
<tr>
<td>Coal</td>
</tr>
<tr>
<td>Coal price</td>
</tr>
<tr>
<td>Gas</td>
</tr>
<tr>
<td>Gas price</td>
</tr>
<tr>
<td>Oil</td>
</tr>
<tr>
<td>Oil price</td>
</tr>
<tr>
<td>Energy</td>
</tr>
<tr>
<td>Energy price</td>
</tr>
<tr>
<td>Output</td>
</tr>
</tbody>
</table>
2.2 Economic changes

The period considered had many different economic phases and went through a period of massive capital investments at the beginning of the sample, as evidenced through an ever-increasing capital share, to a period of upheaval caused by two world wars and a global recession. Table 3 gives an overview of the key developments. One interesting point is that the extreme spike in energy costs in the 1970s and 1980s seemed to have an immediate negative effect on capital returns.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Arith. mean</th>
<th>Geom. mean</th>
<th>Min (Year)</th>
<th>Max (Year)</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross output</td>
<td>340190</td>
<td>223890</td>
<td>46413 (1855)</td>
<td>1156900 (2015)</td>
<td>311270</td>
<td></td>
</tr>
<tr>
<td>Output change</td>
<td>2.0986</td>
<td>2.1828</td>
<td>−13.1 (1921)</td>
<td>11.754 (1940)</td>
<td>3.1723</td>
<td></td>
</tr>
<tr>
<td>Labor</td>
<td>22673</td>
<td>21778</td>
<td>12454 (1855)</td>
<td>35870 (2015)</td>
<td>6217.7</td>
<td></td>
</tr>
<tr>
<td>Capital</td>
<td>522900</td>
<td>305770</td>
<td>63633 (1855)</td>
<td>2421400 (2015)</td>
<td>597640</td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>135.43</td>
<td>122.74</td>
<td>33.977 (1858)</td>
<td>212.7 (1973)</td>
<td>53.43</td>
<td></td>
</tr>
<tr>
<td>Wages</td>
<td>8390.9</td>
<td>6574.8</td>
<td>2233.1 (1858)</td>
<td>22372 (2010)</td>
<td>6028.2</td>
<td></td>
</tr>
<tr>
<td>Labor share</td>
<td>64.138</td>
<td>63.954</td>
<td>52.205 (1871)</td>
<td>72.761 (1932)</td>
<td>4.8069</td>
<td></td>
</tr>
<tr>
<td>Capital share</td>
<td>28.804</td>
<td>28.279</td>
<td>20.03 (1975)</td>
<td>44.002 (1871)</td>
<td>5.6933</td>
<td></td>
</tr>
</tbody>
</table>

The capital share was very high in the beginning of the sample, during the period of rapid industrialization and a fast expansion of the productive capital stock. Additionally, wages in the middle of the 19th century were still very low. At this time, energy prices were also relatively low, which means that the operating costs for the machinery in terms of energy requirements were moderate. The capital stock increased by 2.29% annually during the sample, while the labor force grew only by .66% per year, which means that substantial capital deepening took place. Capital returns fell substantially during the sample. While they reached levels as high as 33% in 1871, they have remained mostly below 19% since the 1970s. Wages grew by 4.36% annually, while energy expenditures grew by 2.30%. Due to the long period considered, the factor shares fluctuated somewhat; however, the overall trend in the labor share is remarkably stable. In contrast to the American economy, where there is persistent evidence for a declining labor share, in the British economy, the labor share was almost constantly above 70% in the period 1935—2015 (it was 70.4% in 1935 and 68.6% in 2015). A peculiar point is that the point of the lowest energy share is not at the beginning of the sample, but rather in 1998, well after the energy crises of the 1970s and 1980s. This demonstrates the structural shift that happened to the British economy between the 1960s and 1980s, when deindustrialization and privatization of state-owned companies were undertaken to revitalize a sclerotic economy. That
means that the economy shifted to industries with an inherently lower demand for energy and capital inputs, a factor that could explain a labor share that is very high compared to that of the US economy (c.f. Frieling and Madlener, 2017).

Table 3 also shows the stark difference between the geometric mean and arithmetic mean for the different production inputs and output. Energy and labor show growth that is almost linear, and therefore feature very similar values for both means, whereas the arithmetic mean of output is 52% larger than its geometric mean. For the capital stock, the difference is even more extreme: the arithmetic mean is 71% larger than the geometric mean, highlighting the substantial capital deepening that happened over time.

In the following section, we describe the aggregate production function that we use to analyze the relationship between the input factors.

3 The aggregate production function

The development of substitution possibilities between factor inputs over long periods is subject to much debate, such as Chirinko (2008), who discusses the different elasticity estimates between labor and capital inputs. The influence of technical change is complicating the estimation of elasticities, as highlighted by Diamond et al. (1978), who theorized that the simultaneous identification of the elasticity and technical change was impossible. Later research, such as Klump et al. (2007), León-Ledesma et al. (2010), Acemoğlu et al. (2012), Henningsen and Henningsen (2012), León-Ledesma et al. (2015), have shown that there are work-arounds to this problem, however. Using the techniques from León-Ledesma et al. (2010), previous work such as Kander and Stern (2014) and Frieling and Madlener (2017) have estimated that even over long periods the elasticity of substitution in CES functions can be below unity. However, CES function parameters are fixed, which could obscure some underlying developments. In particular the question of whether capital and energy should be considered as complements or substitutes has been the subject of discussion, such as Berndt and Wood (1979), or Apostolakis (1990).

For this analysis, we employ a nested, normalized CES model with three input factors, as proposed
by Frieling and Madlener (2016):

\[
\frac{Y}{Y_0} = \psi \left( \alpha_V V^{\frac{(\sigma-1)\sigma_{KL}}{\sigma_{KL}-1}} + \alpha_E \left(e^{\gamma_E (t-t_0)} \frac{QE}{Q_0E_0} \right) \frac{\sigma-1}{\sigma} \right) \tag{2}
\]

with \(V = \left[ \alpha_L \left(e^{\gamma_L (t-t_0)} \frac{L}{L_0} \right)^{\frac{\sigma_{KL}}{\sigma_{KL}-1}} + (1-\alpha_L) \left(e^{\gamma_K (t-t_0)} \frac{K}{K_0} \right)^{\frac{\sigma_{KL}}{\sigma_{KL}-1}} \right].\)

Here, gross output \(Y\) is produced using the input factors capital \((K)\), labor \((L)\), and energy \((E)\). The capital stock and labor inputs are nested in the CES process \(V\), which has the internal elasticity of substitution \(\sigma_{KL}\) and the labor share parameter \(\alpha_L\). \(V\) has the share parameter \(\alpha_V\) and the elasticity of substitution \(\sigma\) with respect to energy. All production factors are assumed to be subject to factor-biased technical change with a linear time trend, which is defined as \(\Gamma_0 e^{\gamma_E (t-t_0)}\). \(\Gamma_0\) is normalized to unity and therefore omitted. As explained in Section 2.1, there is substantial heterogeneity in the energy mix that was used during the sample period. The quality vector \(Q\) accounts for this. \(Q_0\) represents the normalization point of the quality index to the year \(t_0\). We also have the normalization parameter \(\psi\) with \(E[\psi] = 1\).

Following León-Ledesma et al. (2010) we use a system of equations to aid the estimation. The cross-equation constraints and the normalization of the input factors help with the identification of the elasticity and technical change parameters. The system of equations consists of the production function, as well as its derivatives with respect to the three input factors. The identifying assumption is that the input factors are paid according to their marginal productivity\(^3\). This gives us the following system:

\[
\ln \frac{Y}{Y_0} = \ln \psi + \frac{\sigma}{(\sigma-1)} \ln \left( \alpha_V V^{\frac{(\sigma-1)\sigma_{KL}}{\sigma_{KL}-1}} + (1-\alpha_V) \left(e^{\gamma_E (t-t_0)} \frac{QE}{Q_0E_0} \right) \frac{\sigma-1}{\sigma} \right) \tag{3a}
\]

\[
\ln w = \frac{1}{\sigma} \ln \left( \frac{Y}{Y_0} \right) - \frac{1}{\sigma_{KL}} \ln \left( \frac{L}{L_0} \right) + \frac{\sigma_{KL} - 1}{\sigma_{KL}} \ln \left( \gamma_L (t-t_0) \right) + \frac{\sigma-1}{\sigma} \ln \psi + \frac{\sigma - \sigma_{KL}}{\sigma (\sigma_{KL} - 1)} \ln V + \ln \frac{Y_0 \alpha V \alpha L}{L_0} \tag{3b}
\]

\[
\ln r = \frac{1}{\sigma} \ln \left( \frac{Y}{Y_0} \right) - \frac{1}{\sigma_{KL}} \ln \left( \frac{K}{K_0} \right) + \frac{\sigma_{KL} - 1}{\sigma_{KL}} \ln \left( \gamma_K (t-t_0) \right) + \frac{\sigma-1}{\sigma} \ln \psi + \frac{\sigma - \sigma_{KL}}{\sigma (\sigma_{KL} - 1)} \ln V + \ln \frac{Y_0 \alpha V (1-\alpha L)}{K_0} \tag{3c}
\]

\[
\ln p = \frac{\sigma-1}{\sigma} \left( \ln \psi + \gamma_E (t-t_0) \right) + \frac{1}{\sigma} \ln \left( \frac{Y}{Y_0} \right) + \frac{(1-\alpha_V)Y_0}{(Q_0E_0)} + \ln \frac{(Q_0E_0)}{E_0}. \tag{3d}
\]

However, as shown in Frieling and Madlener (2016), estimation of this function can be problematic.

\(^3\)Due to concerns about the data, we refrain from assuming a markup, as it is unlikely that such a markup would be consistent throughout the long time period considered.
Without additional constraints, the non-linear nature of the function and the relatively flat objective function may lead to biased or even spurious results. Since we are interested primarily in the elasticity of energy with respect to the other input factors, we therefore hold $\sigma_{KL}$ fixed at a likely value between 0.5 and 0.8, which reflects the empirical evidence (c.f. Chirinko, 2008; Frieling and Madlener, 2017).

In order to analyze the differences in the development of elasticities and technical change, we will proceed as follows. First, we estimate the model over the whole 161-year sample using likely values for $\sigma_{KL}$ in order to establish a baseline. We then estimate the system over the first 81 years from 1855—1935 and over the second 81 years from 1935—2015, which is a sample size roughly comparable to the one used in Frieling and Madlener (2017). Afterwards, we estimate the system over moving 51 year samples starting with 1855—1905. This provides us with a timeline of changing elasticities and evolving technical change over the sample years. The 51-year sample size is also comparable to other related empirical studies, such as Arrow et al. (1961), Klump et al. (2007), or León-Ledesma et al. (2015).

3.1 Normalization

For the estimation strategy pursued in this paper, normalization is of particular importance, because the estimation is performed over varying sub-samples. This means that we cannot rely on one unique common point of normalization, because it would necessarily lie outside the scope of some of the sub-samples. Instead, the parameters are renormalized for each sample. For the input factors and output, we use the geometric mean in the sub-sample, since these factors exhibit percentage-based growth, as shown by the divergence of their arithmetic and geometric means (see Section 2.2). The quality index and time are normalized to the midpoint of the sample, whereas the share parameters are normalized to the arithmetic mean of the sub-sample.

The normalization of CES functions has become standard for empirical studies, because it has a number of desirable effects, both in terms of the economic interpretation of the parameters, and in terms of helping with the estimation. Normalization converts the output and factor measures to index numbers relative to a given base year. It also has the effect of making the factor inputs unitless, which, in turn, allows the share parameters and productivity parameters to, in turn, become unitless except for the time-dimension. Klump and de la Grandville (2000) show that without normalization, the distribution parameters in a CES function have no clear economic interpretation, as they depend on the elasticity of substitution and the steady-state factor shares. Klump et al.
(2007) explain how normalized CES functions can be used to identify factor-biased technical change, which has been utilized by León-Ledesma et al. (2010) to develop the system of equations approach for estimating the CES functions. Cantore and Levine (2012) show the usefulness of normalization in DSGE modeling. While there are some authors, such as Temple (2012), who are more cautious as to the implications of normalization on comparative statics, Klump et al. (2012) and León-Ledesma et al. (2015) convincingly prove the beneficial effect of normalization for the identification of factor-biased technical change in the context of a CES production function.

This approach does not come without drawbacks, however. As the economy is subject to shocks and outside events, sometimes large deviations from the “steady state” of growth are possible. As is evident from the fluctuations of the factor shares over time, even the assumption of balanced growth seems restrictive and not representative of the economic development. That is, the point of normalization might represent factor ratios that are above or below the optimal factor use or might not even lie precisely on the production function, thus necessitating the inclusion of $\psi$ as a scaling parameter. The use of the geometric mean alleviates this, as we expect the effect of shocks to diminish in a 50-year sample.

4 Empirical results

4.1 The years 1855—2015

| Table 4 |
| Estimation results for 1855—2015, normalized at 1935 |

<table>
<thead>
<tr>
<th>$\sigma_{KL}$</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>0.9695</td>
<td>0.9677</td>
<td>0.9672</td>
<td>0.9676</td>
</tr>
<tr>
<td></td>
<td>(0.0048)</td>
<td>(0.0046)</td>
<td>(0.0045)</td>
<td>(0.0047)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.0239</td>
<td>1.0242</td>
<td>1.0242</td>
<td>1.0246</td>
</tr>
<tr>
<td></td>
<td>(0.0018)</td>
<td>(0.0020)</td>
<td>(0.0021)</td>
<td>(0.0023)</td>
</tr>
<tr>
<td>$\gamma_L$</td>
<td>0.0018</td>
<td>0.0014</td>
<td>0.0008</td>
<td>-0.0004</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0002)</td>
<td>(0.0002)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>$\gamma_K$</td>
<td>-0.0061</td>
<td>-0.0049</td>
<td>-0.0034</td>
<td>-0.0008</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0003)</td>
<td>(0.0004)</td>
<td>(0.0005)</td>
</tr>
<tr>
<td>$\gamma_E$</td>
<td>0.1310</td>
<td>0.1290</td>
<td>0.1279</td>
<td>0.1278</td>
</tr>
<tr>
<td></td>
<td>(0.0016)</td>
<td>(0.0015)</td>
<td>(0.0015)</td>
<td>(0.0015)</td>
</tr>
</tbody>
</table>

D.R.C. 0.0000 0.0000 0.0000 0.0000
(3a) $R^2$ 0.9960 0.9959 0.9958 0.9957
(3b) $R^2$ 0.9681 0.9799 0.9860 0.9896
(3c) $R^2$ -0.6343 -0.0578 0.2712 0.4707
(3d) $R^2$ 0.8217 0.8217 0.8218 0.8217

HAC standard errors in parentheses.
We first estimate the system over the whole sample using a GMM estimator with the lagged factor inputs and output as instruments. The results are reported in Table 4. We follow Frieling and Madlener (2017) and use four likely values 0.5, 0.6, 0.7, and 0.8 for $\sigma_{KL}$ that are motivated by previous research which strongly points to below-unitary substitution between capital and labor (Chirinko, 2008). With this we can observe a number of phenomena already mentioned in Section 3. First, we see that the estimated values of $\psi$ are all smaller than one, implying that the normalization point might represent a significant departure from the steady state. $\sigma$ is very consistently estimated to be slightly above unity, making it a gross substitute in the observed sample. A result of this is the lack of clear identification of energy-specific factor change. Since the CES function has a discontinuity around $\sigma = 1$, where it approaches the Cobb-Douglas function, technical change parameter estimates can become inconsistent. In the Cobb-Douglas case, there is no factor-specific technical change, because it is a multiplicative function. This means that we likely have positive energy-specific technical change, but its size remains unclear.

It is interesting, however, that just like in Frieling and Madlener (2017) for the US, the capital-specific technical change parameter is negative. The $R^2$ for eq. (3c) only becomes positive for $\sigma_{KL} \geq 0.7$, which indicates that the capital-labor elasticity also approaches the Cobb-Douglas case in the very long run. However, the generally low $R^2$ values for eq. (3c) point to a general problem that the development of the capital stock and the user cost of capital cannot properly be captured in the
system of equation over such a long sample. This is also evident from the residuals\(^4\), as can be seen in Figure 1. These highlight the numerous shocks that affected the different input factors and the economy as a whole. If the sample is split into two periods, we see that the results are more in line with other studies, and we see that estimation over the whole sample obscures the effect of the developments during this period.

**4.2 The years 1855—1935**

We now repeat the procedure described in Section 4.1 on the first half of the sample only, i.e. the years 1855—1935. The function is again normalized using the geometric mean of the output and input variables over this sub-sample, the average factor shares during this time. Additionally, the quality index is normalized to be 1 at time \(t_0\) in 1895. The system is then estimated using the same four values for \(\sigma_{KL}\).

<table>
<thead>
<tr>
<th>Table 5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Estimation results for 1855—1935, normalized at 1895</strong></td>
</tr>
<tr>
<td>(\sigma_{KL}) &amp; 0.5 &amp; 0.6 &amp; 0.7 &amp; 0.8</td>
</tr>
<tr>
<td>(\psi) &amp; 1.0122 &amp; 1.0138 &amp; 1.0156 &amp; 1.0165 &amp; (0.0023) &amp; (0.0023) &amp; (0.0023) &amp; (0.0024)</td>
</tr>
<tr>
<td>(\sigma) &amp; 0.6943 &amp; 0.7154 &amp; 0.7467 &amp; 0.7634 &amp; (0.0231) &amp; (0.0232) &amp; (0.0244) &amp; (0.0285)</td>
</tr>
<tr>
<td>(\gamma_L) &amp; 0.0041 &amp; 0.0035 &amp; 0.0024 &amp; -0.0001 &amp; (0.0002) &amp; (0.0002) &amp; (0.0002) &amp; (0.0003)</td>
</tr>
<tr>
<td>(\gamma_K) &amp; 0.0049 &amp; 0.0074 &amp; 0.0111 &amp; 0.0177 &amp; (0.0001) &amp; (0.0002) &amp; (0.0003) &amp; (0.0005)</td>
</tr>
<tr>
<td>(\gamma_E) &amp; -0.0182 &amp; -0.0194 &amp; -0.0217 &amp; -0.0207 &amp; (0.0014) &amp; (0.0016) &amp; (0.0019) &amp; (0.0021)</td>
</tr>
<tr>
<td>D.R.C. &amp; 0.0000 &amp; 0.0000 &amp; 0.0000 &amp; 0.0000</td>
</tr>
<tr>
<td>(3a) (R^2) &amp; 0.9767 &amp; 0.9806 &amp; 0.9846 &amp; 0.9889</td>
</tr>
<tr>
<td>(3b) (R^2) &amp; 0.9327 &amp; 0.9444 &amp; 0.9534 &amp; 0.9624</td>
</tr>
<tr>
<td>(3c) (R^2) &amp; 0.7616 &amp; 0.8036 &amp; 0.8266 &amp; 0.8425</td>
</tr>
<tr>
<td>(3d) (R^2) &amp; 0.5580 &amp; 0.5696 &amp; 0.5841 &amp; 0.5849</td>
</tr>
</tbody>
</table>

HAC standard errors in parentheses.

Table 5 shows the results of this estimation. The values for \(\psi\) are now closer to what we expect, and are very close to unity. Similarly, \(\sigma\) is consistently estimated to be below unity, around 0.7. We also see very different results when it comes to the technical change parameters, as energy-specific technical change is actually negative during this period, i.e. technical change was actually energy-using rather than energy-saving. The fit of the model is also better, as can be seen by the fact that all four specifications give comparable \(R^2\) statistics.

\(^4\)The residuals are not plotted for all \(\sigma_{KL}\) values to reduce visual clutter.
In general, this period of rapid expansion of energy consumption is marked by productivity increases for capital and labor, while the estimation results for energy-augmenting technical change are negative. Only when $\sigma_{KL} \geq 0.8$ is there doubt about the presence of labor-augmenting technical change. This highlights the difficulty of identifying the exact direction of technical change. The degree of complementarity between any two input factors depends critically also on the relevant elasticity of substitution with respect to the third input factor. The estimates for $\gamma_K$ are in line with expectations for a period of rapid innovation of the capital stock and a relative increase in the availability of affordable energy services. Among other studies, Fouquet and Pearson (2006) point to the constraints imposed by the high costs of certain energy services. During this period, the quality index of energy inputs doubled, reflecting the shift in the energy mix from being purely coal-based to incorporating other fuels, such as oil. Additionally, the British economy was not as labor-starved during this period, since a persistent trend towards urbanization led to the availability of relatively low cost labor in the urban centers. The result is also in line with those of Kander and Stern (2014) who look at the Swedish economy for the years 1850—1950.

The residuals depicted in Figure 2 are also more closely clustered around zero than those reported in Section 4.1. We see that the overall fit is already improved when the sample is halved.
4.3 The years 1935—2015

Next, we repeat the estimation for the second half of the sample, from 1935—2015. Here, a completely different picture emerges. In this case, the midpoint of the sample $t_0$ is 1975, which falls in a period of economic turmoil and transition for the British economy. Again, we see from the values of $\psi$ around unity and the generally higher $R^2$ values that the fit over this shorter period is better than over the whole sample, only in this case it is the energy price series, which has the worst fit. As Klump et al. (2012) note, the non-linearity of the CES function means that some divergence for unity is to be expected. Over the whole sample, even incremental changes to the structure of the economy accumulate. This means that the rigidity of the model cannot accurately reflect the development of the elasticity of substitution and factor-biased technical change. When looking at the results reported in Table 6, we immediately see that the estimates for $\sigma$ are considerably higher, and are all, with the exception of $\sigma_{KL} = 0.5$, above unity. This means that energy is a gross substitute during this period.

The results reflect the overall changes in the British economy during this period. The coal industry began to decline in the 1960s. The global economy in the 1970s was flagging as a consequence of the 1973 oil crisis. A barely recovered British economy was hit first by widespread strikes in the "Winter of Discontent" of 1978/1979 and then by a second oil crisis. The result was a massive political push to privatize state-owned companies and a shift between and within sectors towards less energy-intensive...
products (Jenne and Cattell, 1983). As a result, the energy intensity of output fell considerably. The estimation results reflect this development quite well. As described in Section 2.2, 1975 was the year with the lowest capital share, as well as only two years after the maximum total energy consumption. The preceding oil crisis was a massive exogenous shock to the economy, which further aggravated the crisis of British manufacturing.

The results highlight that longer sample sizes can hide substantial heterogeneity in the CES function. We see that the two results — where energy is either a complement or substitute — paint a very different picture of the structure of the UK economy a result that is not evident when only the whole sample is considered. The second half of the sample seems to have experienced greater shocks than the first half, in particular for energy prices, as is evident from Figure 3. This might be the result of an increasing dependence on foreign imports. In 1855, almost all energy demand could be met by domestic coal production, while on the eve of the first oil crisis in 1973, the import dependency for fuels was 52.2%.

4.4 Comparison with the US

The question that is raised by the results in Section 4.3, is whether what we observe is typical and generalizable for western industrialized countries. The sample in Section 4.3 is very similar in time to the one used in Frieling and Madlener (2017) for the US, which allows us to compare the
developments in both countries, using similar methodologies. The results of Frieling and Madlener (2017) are reported in Table 7. There are some caveats to the comparison, however. First, in Frieling and Madlener (2017), capital stock and capital income from housing are included in the analysis, which could affect the results regarding capital-saving technical change. Second, we include a quality index to reflect the shift in the energy mix of the UK. Finally, in Frieling and Madlener (2017), output is just defined as GDP and does not add energy expenditures to it. However, despite these methodological differences, the extent of the divergence of the results is surprising.

### Table 7

<table>
<thead>
<tr>
<th>$\sigma_{KL}$</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>0.99</th>
<th>1.01</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0231</td>
<td>1.0280</td>
<td>1.0317</td>
<td>1.0349</td>
<td>1.0398</td>
<td>0.9964</td>
<td>0.967</td>
<td>1.099</td>
<td>1.057</td>
<td>0.989</td>
</tr>
<tr>
<td></td>
<td>(0.0059)</td>
<td>(0.0057)</td>
<td>(0.0055)</td>
<td>(0.0054)</td>
<td>(0.0056)</td>
<td>(0.0097)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.007)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.6348</td>
<td>0.6381</td>
<td>0.6469</td>
<td>0.6621</td>
<td>0.7074</td>
<td>1.0024</td>
<td>1.698</td>
<td>0.724</td>
<td>0.983</td>
<td>0.999</td>
</tr>
<tr>
<td></td>
<td>(0.0267)</td>
<td>(0.0256)</td>
<td>(0.0246)</td>
<td>(0.0241)</td>
<td>(0.0246)</td>
<td>(0.0036)</td>
<td>(0.108)</td>
<td>(0.011)</td>
<td>(0.005)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$\gamma_L$</td>
<td>0.0131</td>
<td>0.0136</td>
<td>0.0142</td>
<td>0.0150</td>
<td>0.0162</td>
<td>0.0083</td>
<td>0.023</td>
<td>0.122</td>
<td>-0.029</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.0003)</td>
<td>(0.0004)</td>
<td>(0.0004)</td>
<td>(0.0009)</td>
<td>(0.0001)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\gamma_K$</td>
<td>-0.0034</td>
<td>-0.0027</td>
<td>-0.0020</td>
<td>-0.0015</td>
<td>-0.0016</td>
<td>-0.0134</td>
<td>-0.006</td>
<td>-0.184</td>
<td>0.155</td>
<td>0.003</td>
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<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0003)</td>
<td>(0.0003)</td>
<td>(0.0003)</td>
<td>(0.0011)</td>
<td>(0.001)</td>
<td>(0.006)</td>
<td>(0.015)</td>
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<tr>
<td>$\gamma_E$</td>
<td>0.0031</td>
<td>0.0025</td>
<td>0.0017</td>
<td>0.0005</td>
<td>-0.0027</td>
<td>0.4007</td>
<td>0.039</td>
<td>0.003</td>
<td>-0.642</td>
<td>0.445</td>
</tr>
<tr>
<td></td>
<td>(0.0034)</td>
<td>(0.0035)</td>
<td>(0.0036)</td>
<td>(0.0039)</td>
<td>(0.0048)</td>
<td>(0.0305)</td>
<td>(0.005)</td>
<td>(0.004)</td>
<td>(0.126)</td>
<td>(0.030)</td>
</tr>
</tbody>
</table>

D.R.C. 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

(3b) $R^2$ 0.6635 0.6651 0.6665 0.6680 0.6703 0.6808 0.683 0.666 0.664 0.583
(3c) $R^2$ 0.5874 0.8091 0.8756 0.8922 0.8857 0.7930 0.673 0.868 0.791 0.747
(3d) $R^2$ 0.9519 0.9659 0.9722 0.9762 0.9814 0.9923 0.993 0.992 0.993 0.993

There are similar developments when it comes to labor- and capital-specific technical change, e.g. in that both economies saw mostly labor-saving technical change during this period and had moderately negative capital-specific technical change. In the US, the observed energy-specific technical change is not statistically significant, while it is estimated to be slightly negative in the UK. The difference is whether energy can be considered to be a gross substitute or a gross complement. In the US, it is clearly a gross complement, while in the UK, it is clearly a gross substitute. Why the results are so different deserves some explanation.

For one, the US economy was less affected by the oil crises of the 1970s and 1980s, and its industrial base was able to better adapt to the post-WWII economy. In the US, the share of energy expenditures never went above 10%, which was in 1980. The average energy share in the US was just above 3%, while for the UK energy expenditures were on average 7.1% of total output (Frieling and Madlener, 2017). Due to the rush for gas in the UK and an almost complete disappearance of coal from the energy mix, the quality index is almost three times as large in 2015 as in 1935, and almost six times larger than in 1855. This captures part of the efficiency gains that are possible by shifting towards
a more efficient and economical fuel input.

One other possible reason for the divergent development was the greater reliance of the UK on coal, which became more expensive as domestic production ceased to be economically viable. The shift to other energy carriers necessitated investments which increased the constraints on the available capital, a fact that was exacerbated by the exploding energy costs in the 1970s. While most industrialized countries went through an economic slump as a consequence of the oil crises, the difference might have been that the burden of replacing the aging and obsolete capital stock led to widespread bankruptcies. Turnheim and Geels (2012) highlight the wide range of social, economic, and political factors that arrayed themselves to destabilize the coal industry in particular, which had already lost much of its status as the backbone of the British industry through the implementation of measures such as the 1956 Clean Air Act. While high oil prices led to a short period of renewed competitiveness for the coal industry, the market reforms of the Thatcher government and the withdrawal of state subsidies dashed the hopes of a coal revival. This means that during this time, many companies went bankrupt that could have survived in an environment like the US. The results thus reflect the type of companies that survived in the aftermath of the oil crises.

The remarkable effect of the energy crises on British industrial production was immediately apparent in the aftermath. Already Jenne and Cattell (1983) argue that a sizable portion of the observed increases in the efficiency of energy use, as measured in energy use per unit of output, can be attributed to structural change between and within sectors. The effect is magnified when output is measured in value as opposed to quantity, because the prices of imports and agricultural products rose sharply in the 1970s. They also highlighted that the first recession of 1974 and 1975 led to measures like the three-day week and efforts to keep companies afloat with reduced load factors, while the second recession of the 1980s led to a complete market exit of many firms. This is the reason why overall fossil fuel consumption peaked in 1973 for the UK, whereas it peaked in 2007 in the US, where the oil crises had led to just a short dip in energy consumption. The crises had still had an effect, and energy consumption in the US declined by roughly 10% between 1979 and 1983. However, this was hardly a permanent reversal of the trend, unlike the one we see in the UK. After 1983 energy consumption steadily increased again, reaching its pre-crisis levels in 1988, and continuing to increase until the beginning of the great recession in 2007. Together, the results presented in Sections 4.1, 4.2, and 4.3 show an economy in transition. They demonstrate that increasing the sample size does not always improve the estimation of the production function for a single economy, especially if there were significant changes in the production function during that time.
5 Energy in the British economy over time

In order to better monitor how the changes manifested themselves in the production function, we now estimate the system again, but over a smaller sample. We create 111 sub-samples, each spanning 51 years, starting from 1855—1905 until 1965—2015. We then estimate the normalized system of equations over each of the sub-samples and track the changes in the parameter estimates as the estimated sample changes. This allows us to capture the changes that happened in the medium run. The sample size is picked to be both representative of a typical sample size for macroeconomic studies and to provide a large enough sample for a reliable estimation. The function is normalized over the respective sub-samples with the same procedure as in Section 4. In the following graphs, the results are reported on the time axis using the respective normalization point $t_0$ as a reference point, since this is the point around which the sample is centered. Figure 4 shows a smoothed curve of the average estimated function parameters. Standard errors are shaded. Figure 6 depicts the estimated coefficient for the technical change parameter separated by $\sigma_{KL}$ value, as well as the smoothed average. The detailed estimation results for twelve of the samples can be found in Appendix A.

![Figure 4](image)

**Figure 4**

_Estimation coefficients over time_

The most striking result that is evident from Figure 4 is the remarkable way in which the elasticity $\sigma$ developed over time. We see that the status quo is that $\sigma$ is generally below unity, indicating that
energy is a gross complement to other input factors. This seemingly constant result is then called into question in the samples centered around 1955—1980, where the elasticity of substitution of energy suddenly rapidly increases like a wave, before it drops down to values below unity again. Figure 5 shows this result in more detail, indicating that the precise moment where $\sigma$ becomes greater than unity also depends slightly on $\sigma_{KL}$. Nevertheless, the result is noteworthy because of the implications drawn by Klump and de la Grandville (2000), who show that increasing the elasticity of substitution should enable growth. Interestingly, the periods of high energy-specific technical change are first accompanied by a relatively low elasticity of substitution, while the second peak is accompanied by a high elasticity of substitution.

A likely explanation is that the flexibility which characterizes a production function with a high elasticity of substitution comes with additional costs. That means that a period characterized by repeated exogenous shocks to energy prices warrants this premium and can lead to a renewed focus on finding the most productive uses for energy. Additionally, other research about the development of the makeup of British industry in the aftermath of the 1970s, such as Jenne and Cattell (1983), shows that there has been a tremendous pressure on firms with high energy costs to exit the market completely. When we assume that there was an increased rate of exit for firms with low substitution possibilities and high energy intensities, and a comparatively higher rate of market entry for firms that do not rely on energy inputs as much, it would explain the sudden increase in the elasticity of substitution.
substitution that we observe. In this way, the high elasticity of substitution and the high rate of energy-saving technical change complement each other and facilitate a greater energy independence in the form of tertiarization that was more pronounced than in other countries, such as the US, Japan, or Germany. Similarly, this imbalance in the relative rate of market entry and exit cannot be permanent. As the number of remaining energy-intensive companies dwindles, the high elasticity of substitution decreases rapidly again.

Figures 4 and 6 also show the sweeping changes of a linear time trend of factor-biased technical change that happened over the different periods. In particular energy inputs seem to have been the target of ever increasing factor-saving technical change. We see in Figure 6 that there are two distinct waves of energy-saving technical change. The first appears to be the result of the improvements happening in the wake of industrialization, when the use of new types of energy spread rapidly through the economy and the increased demand led to first price spikes and worries about the ability of the supply to keep up with a rapidly increasing demand for energy. This coincides with the period of rapid productivity increases thought to be the result of a shift in production technology. Widespread electrification did not initially increase industrial productivity by as much as was hoped,

\[^5\text{This development was foreseen by Jevons (1865), who worried about the effects of a depletion of coal reserves on the competitiveness of the British economy.}\]
because the organization of factories was driven by the demands arising from steam engines. Steam engines required central drive shafts around which production was organized, because this enabled substantial scale effects and efficiency gains, since steam engines are much more efficient the larger they are. Initially, however, the switch to electric motors did not change this paradigm. Only after production facilities were designed to account for the flexibility and modularity that was possible with the use of many smaller electric motors (as opposed to a central massive steam engine) did productivity pick up. This led to the large increases in total factor productivity that were observed during the 1920s\(^6\).

The second burst of energy-specific productivity increases, ironically, seems to be centered around the energy crises of the 1970s and 1980s, when the British economy began to deindustrialize. During this period, the shift from an energy-intensive heavy industry towards a service-oriented economy led to a substitution away from capital- and energy-intensive manufacturing towards output that requires more skilled workers. This second burst coincided with a substantial increase in the elasticity estimates as well, as can be seen in Figure 4. In general, Figure 6 shows how much factor-specific technical change is affected by periods of economic transition.

It is interesting that WWII did not seem to have a very large effect on the development of the elasticity and technical change parameters. One possible reason could be that the public sector and wartime economic policies had a stabilizing effect. While economic activity was certainly impacted, the UK was during this time still largely reliant on coal and had sufficient domestic capacity to ensure the satisfaction of basic energy demand. Industrial capacity was needed for the production of wartime necessities, ensuring that there could be continuity in the industrial base. Additionally, there was not a lot of widespread destruction of the industrial base in the UK. In fact, Figure 6 actually points toward a relative stagnation of energy-saving technical change during the wartime years. Rather, most technical change was labor-saving, which points to an acute labor shortage during this time. In the aftermath of the war, energy-saving technical change became again the priority.

The status quo, however, seems to be a complementarity between all three input factors. This analysis is primarily concerned with the substitution possibilities for energy, whereas other nesting arrangements of eq. (2) are possible as well. In fact, there is a large literature that is primarily concerned with the question of whether capital and energy are complements or substitutes when nested together. While this methodology does not provide answers to this specific question, the other two nesting possibilities of (KE)L and (LE)K also show complementarity between the nests

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\(^6\)An overview of the delayed impact of electrification on industrial productivity can be found in David (1990).

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and the outer input factor\(^7\).

That the elasticity change comes as a wave that subsides again provides some clues as to the limits on endogenously increased substitution elasticities. While there is likely some non-zero value in flexibility leading to elasticity parameters somewhere between zero and unity, increasing this elasticity is likely costly both in terms of capital investments and in operating efficiency. In fact, the high elasticity estimates we see during the 1970s and 1980s are also the result of a complete exit of the firms that are most affected by the sudden increase in energy prices from the market. Contrary to the US, the UK saw significant restructuring of whole sectors of the economy, a process that was seen as an overdue and necessary adjustment to, in particular, a changing energy environment.

We might also see a similar development that mirrors the role provender played at the beginning of the 20th century. The energy service that was required in certain sectors, notably the agrarian sector, could only be delivered via means that utilized expensive and inefficient energy sources. This meant buying feed for draft animals even as more affordable fuels replaced muscle power elsewhere in the economy. A similar phenomenon could explain the decreasing elasticity of substitution that we see today: the energy services that are most reliant on fossil fuel inputs today are those that are least able to switch to other fuel sources, such as renewable energy, or are crucial to the economy. Transportation, for example, is a sector that relies almost exclusively on oil as a fuel. It is also a sector that enables other sectors to reduce their energy dependency, for example through the incorporation of a global supply chain. Yet, until there is substantial electrification of the transport sector, there is little that can be done to substitute away from oil in a meaningful way.

6 Conclusions

The input factors energy, labor, and capital can generally be considered as gross complements. Our analysis shows that during the first 100 years of our sample there was remarkable stability in the parameters of an estimated CES function with respect to the elasticity of substitution, albeit less so with technical change. However, this status quo can be disrupted. In the case of the UK, we see that the period of economic turmoil between the 1960s and the early 1980s, which was precipitated by a crisis of the British coal industry and the following two oil crises, led to fundamental changes in the makeup of the economy. During this transitional period, we see a dramatic increase in the elasticity of substitution of energy and energy productivity. This was partly the result of an exit of inefficient

\(^7\)For confirmation, the analysis was also performed with data for the US, where we also found general complementarity.
and inflexible firms, and partly the result of between- and within-sector shifts towards more flexible and efficient production. At the end of this period, the British economy was transformed. However, we find that the increase in elasticity is not persistent. Rather, we see a reversion to elasticity values even lower than before the transition, to just 0.3. This points towards a high cost for preserving the flexibility that is the foundation of a larger-than-unity elasticity of substitution.

Just as the agricultural sector was still reliant on provender as an energy source in the 1900s because other energy services could not fulfill the demands of the sector, the remaining demand for fossil fuels is even less flexible than before the transition. Since we do not account for electricity that is directly derived from renewable sources or nuclear fuel, the remaining energy demand we can observe is the demand that is most reliant on oil and gas. This means that the elasticity of substitution in the modern British economy is actually lower than during the Industrial Revolution. This should put a damper on the hopes that elasticity increases can be a permanent foundation for economic growth, or a foundation of environmental protection and other policy goals. Rather, it seems like deviations from the norm of complementarity are the result of an economy under tremendous stress.

The results in this paper should also serve to caution researchers about the dramatic effects of sample selection on results. Forecasting and model calibration often assume the out-of-sample validity of empirical results. In our case, the results from an 81-year sample spanning from 1935–2015 were dominated by the massive changes that happened over a 25-year period in the middle of the sample. Similarly, even a 100-year sample from 1855–1955 would not have been sufficient to capture and predict the changes that were waiting to happen.

The British economy has changed dramatically over the last 160 years and can be expected to change further in the wake of Brexit. Energy was as much a spark for its modernization and industrialization as it was the millstone dragging it down again. The expansion of the availability of energy services that was made possible by the Industrial Revolution was accompanied by rapid growth as well as productivity increases for energy and other inputs. With the exhaustion of domestic coal mines and an increasing dependence on foreign energy sources came a period of decline, which was exacerbated by energy shocks that hit the British economy harder than other countries such as the US. The economy underwent changes to increase competitiveness and divested itself from industrial production. This severely reduced the reliance on energy imports and led to a relatively steady decline in fossil fuel consumption beginning in 1973.

A comparison of the developments in the UK and US over the last 80 years highlights the dramatic changes that the British economy has undergone. While the US economy has shown overall stability,
the results for the UK show that this stability is anything but permanent. The results also indicate that, going forward, energy policy will have to take into account an ever-decreasing scope for substitution for the remaining uses of fossil fuel. For example, even if breakthroughs in battery technology or hydrogen production from renewables would allow the transport sector to switch fuels, the experience of the first wave of electrification has shown that there can be substantial lag time between the development of a new technology and the point at which it actually affects the structure of an economy in a meaningful way. This means that energy consumption is relatively immutable in the short term, which is disheartening in light of the ambitious environmental goals set by governments all over the world.

References


Diamond, Peter, Daniel McFadden, and Miguel Rodriguez, “Measurement of the Elasticity of Factor Substitution and Bias of Technical Change,” in Melvyn Fuss and Daniel McFadden, eds.,


A  Estimation results for selected samples

### Table 8
Estimation results for 1855—1905, normalized at 1880

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<thead>
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### Table 9
Estimation results for 1865—1915, normalized at 1890

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### Table 10

*Estimation results for 1875—1925, normalized at 1900*

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*Estimation results for 1885—1935, normalized at 1910*

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*Estimation results for 1895—1945, normalized at 1920*

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</table>

(3a) $R^2$ | 0.9210 | 0.9106 | 0.9148 | 0.9165 |
(3b) $R^2$ | 0.9506 | 0.9041 | 0.9121 | 0.9232 |
(3c) $R^2$ | 0.7549 | 0.7949 | 0.8140 | 0.8206 |
(3d) $R^2$ | 0.7490 | 0.7338 | 0.7350 | 0.7368 |

### Table 13

*Estimation results for 1905—1955, normalized at 1930*

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<td>0.0021</td>
<td>0.0020</td>
<td>0.0017</td>
</tr>
<tr>
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<td>0.8169</td>
<td>0.7809</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.0057</td>
<td>0.0205</td>
<td>0.0205</td>
<td>0.0165</td>
</tr>
<tr>
<td>$\gamma_L$</td>
<td>-0.0019</td>
<td>0.0069</td>
<td>0.0071</td>
<td>0.0062</td>
</tr>
<tr>
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<td>0.0002</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\gamma_K$</td>
<td>0.0031</td>
<td>0.0128</td>
<td>0.0159</td>
<td>0.0214</td>
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<td>0.0002</td>
<td>0.0003</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\gamma_E$</td>
<td>0.1398</td>
<td>-0.0215</td>
<td>-0.0271</td>
<td>-0.0183</td>
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<td>0.0019</td>
<td>0.0020</td>
<td>0.0029</td>
<td>0.0019</td>
</tr>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

(3a) $R^2$ | 0.9389 | 0.9106 | 0.9155 | 0.9276 |
(3b) $R^2$ | 0.9506 | 0.9041 | 0.9121 | 0.9232 |
(3c) $R^2$ | 0.7549 | 0.7949 | 0.8140 | 0.8206 |
(3d) $R^2$ | 0.7490 | 0.7338 | 0.7350 | 0.7368 |

32
Table 14
Estimation results for 1915—1965, normalized at 1940

<table>
<thead>
<tr>
<th>$\sigma_{KL}$</th>
<th>0.5</th>
<th>0.6</th>
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<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi$</td>
<td>0.9917</td>
<td>0.9932</td>
<td>0.9946</td>
<td>0.9961</td>
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<tr>
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<td>0.0044</td>
<td>0.0040</td>
<td>0.0038</td>
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<td>0.6970</td>
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<td>0.7824</td>
</tr>
<tr>
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<td>0.0389</td>
<td>0.0379</td>
<td>0.0362</td>
<td>0.0337</td>
</tr>
<tr>
<td>$\gamma_L$</td>
<td>0.0135</td>
<td>0.0147</td>
<td>0.0164</td>
<td>0.0190</td>
</tr>
<tr>
<td>s.e.</td>
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<td>0.0005</td>
<td>0.0004</td>
<td>0.0003</td>
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<td>$\gamma_K$</td>
<td>0.0052</td>
<td>0.0057</td>
<td>0.0061</td>
<td>0.0061</td>
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<tr>
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<td>0.0002</td>
<td>0.0002</td>
<td>0.0006</td>
</tr>
<tr>
<td>$\gamma_E$</td>
<td>-0.0294</td>
<td>-0.0322</td>
<td>-0.0367</td>
<td>-0.0453</td>
</tr>
<tr>
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<td>0.0016</td>
<td>0.0021</td>
<td>0.0029</td>
<td>0.0045</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D.R.C.</th>
<th>0.0000</th>
<th>0.0000</th>
<th>0.0000</th>
<th>0.0000</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3a) $R^2$</td>
<td>0.9311</td>
<td>0.9408</td>
<td>0.9496</td>
<td>0.9554</td>
</tr>
<tr>
<td>(3b) $R^2$</td>
<td>0.7699</td>
<td>0.8116</td>
<td>0.8472</td>
<td>0.8853</td>
</tr>
<tr>
<td>(3c) $R^2$</td>
<td>0.5667</td>
<td>0.7287</td>
<td>0.8090</td>
<td>0.8475</td>
</tr>
<tr>
<td>(3d) $R^2$</td>
<td>0.8119</td>
<td>0.8239</td>
<td>0.8366</td>
<td>0.8497</td>
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</tbody>
</table>

Table 15
Estimation results for 1925—1975, normalized at 1950

<table>
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<th>$\sigma_{KL}$</th>
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<th>0.6</th>
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<tr>
<td>$\Psi$</td>
<td>0.9943</td>
<td>0.9953</td>
<td>0.9962</td>
<td>0.9976</td>
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<tr>
<td>s.e.</td>
<td>0.0014</td>
<td>0.0013</td>
<td>0.0012</td>
<td>0.0012</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.6104</td>
<td>0.6318</td>
<td>0.6635</td>
<td>0.7111</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.0097</td>
<td>0.0102</td>
<td>0.0111</td>
<td>0.0129</td>
</tr>
<tr>
<td>$\gamma_L$</td>
<td>0.0170</td>
<td>0.0182</td>
<td>0.0199</td>
<td>0.0227</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0003</td>
</tr>
<tr>
<td>$\gamma_K$</td>
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<td>0.0004</td>
<td>0.0002</td>
<td>-0.0011</td>
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<tr>
<td>s.e.</td>
<td>0.0003</td>
<td>0.0004</td>
<td>0.0005</td>
<td>0.0007</td>
</tr>
<tr>
<td>$\gamma_E$</td>
<td>-0.0311</td>
<td>-0.0329</td>
<td>-0.0360</td>
<td>-0.0419</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0015</td>
<td>0.0022</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>D.R.C.</th>
<th>0.0000</th>
<th>0.0000</th>
<th>0.0000</th>
<th>0.0000</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3a) $R^2$</td>
<td>0.9575</td>
<td>0.9677</td>
<td>0.9775</td>
<td>0.9860</td>
</tr>
<tr>
<td>(3b) $R^2$</td>
<td>0.9282</td>
<td>0.9446</td>
<td>0.9550</td>
<td>0.9649</td>
</tr>
<tr>
<td>(3c) $R^2$</td>
<td>0.1042</td>
<td>0.4947</td>
<td>0.6901</td>
<td>0.7896</td>
</tr>
<tr>
<td>(3d) $R^2$</td>
<td>0.9151</td>
<td>0.9200</td>
<td>0.9259</td>
<td>0.9322</td>
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### Table 16

*Estimation results for 1935—1985, normalized at 1960*

<table>
<thead>
<tr>
<th>$\sigma_{KL}$</th>
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<th>0.8</th>
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<tr>
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<td>0.9877</td>
<td>0.9868</td>
<td>0.9864</td>
<td>0.9869</td>
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<td>0.0019</td>
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<td>$\sigma$</td>
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<td>1.1050</td>
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<td>s.e.</td>
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<td>0.0029</td>
<td>0.0032</td>
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<td>$\gamma_L$</td>
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<td>0.0072</td>
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<td>0.0001</td>
<td>0.0001</td>
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<tr>
<td>$\gamma_K$</td>
<td>-0.0172</td>
<td>-0.0172</td>
<td>-0.0178</td>
<td>-0.0197</td>
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<tr>
<td>s.e.</td>
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<td>0.0003</td>
<td>0.0004</td>
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<tr>
<td>$\gamma_E$</td>
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<td>0.1206</td>
<td>0.1129</td>
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<td>0.0013</td>
<td>0.0014</td>
<td>0.0016</td>
<td>0.0022</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>(3a) $R^2$</th>
<th>(3b) $R^2$</th>
<th>(3c) $R^2$</th>
<th>(3d) $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.R.C.</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>(3a) $R^2$</td>
<td>0.9887</td>
<td>0.9887</td>
<td>0.9889</td>
<td>0.9893</td>
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<tr>
<td>(3b) $R^2$</td>
<td>0.9832</td>
<td>0.9889</td>
<td>0.9915</td>
<td>0.9926</td>
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<tr>
<td>(3c) $R^2$</td>
<td>0.5912</td>
<td>0.7371</td>
<td>0.8016</td>
<td>0.8282</td>
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<tr>
<td>(3d) $R^2$</td>
<td>0.9461</td>
<td>0.9460</td>
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</table>

### Table 17

*Estimation results for 1945—1995, normalized at 1970*

<table>
<thead>
<tr>
<th>$\sigma_{KL}$</th>
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<tr>
<td>$\Psi$</td>
<td>1.0286</td>
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<td>0.0012</td>
<td>0.0010</td>
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<tr>
<td>$\sigma$</td>
<td>0.9570</td>
<td>2.4481</td>
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<tr>
<td>$\gamma_L$</td>
<td>0.0066</td>
<td>0.0218</td>
<td>0.0222</td>
<td>0.0229</td>
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<tr>
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<td>0.0001</td>
<td>0.0002</td>
<td>0.0002</td>
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<td>$\gamma_K$</td>
<td>-0.0195</td>
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<td>-0.0115</td>
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<tr>
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<td>0.0001</td>
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<td>0.0004</td>
</tr>
<tr>
<td>$\gamma_E$</td>
<td>0.1197</td>
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<tr>
<td>s.e.</td>
<td>0.0020</td>
<td>0.0006</td>
<td>0.0008</td>
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</table>

<table>
<thead>
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<th></th>
<th>(3a) $R^2$</th>
<th>(3b) $R^2$</th>
<th>(3c) $R^2$</th>
<th>(3d) $R^2$</th>
</tr>
</thead>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
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<td>0.9911</td>
<td>0.9825</td>
<td>0.9827</td>
<td>0.9845</td>
</tr>
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<td>0.9770</td>
<td>0.9259</td>
<td>0.9608</td>
<td>0.9812</td>
</tr>
<tr>
<td>(3c) $R^2$</td>
<td>0.8861</td>
<td>0.7767</td>
<td>0.7936</td>
<td>0.7993</td>
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<tr>
<td>(3d) $R^2$</td>
<td>0.1749</td>
<td>0.3556</td>
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</table>
Table 18
Estimation results for 1955—2005, normalized at 1980

<table>
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<td>0.0014</td>
<td>0.0015</td>
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<td>0.9210</td>
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<tr>
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<td>0.0012</td>
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</table>

D.R.C. 0.0000 0.0000 0.0000 0.0000

(3a) $R^2$ 0.9830 0.9787 0.9886 0.9818
(3b) $R^2$ 0.9481 0.9498 0.9693 0.9554
(3c) $R^2$ 0.7954 0.8069 0.8111 0.8053
(3d) $R^2$ 0.0000 -0.0145 -0.1696 0.0052

Table 19
Estimation results for 1965—2015, normalized at 1990

<table>
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<th>$\sigma_{KL}$</th>
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<th>0.8</th>
</tr>
</thead>
<tbody>
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<td>1.0070</td>
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<td>0.0007</td>
<td>0.0007</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.2689</td>
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<td>0.2716</td>
<td>0.2725</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.0043</td>
<td>0.0037</td>
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<td>0.0021</td>
</tr>
<tr>
<td>$\gamma_L$</td>
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<td>0.0005</td>
</tr>
<tr>
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<tr>
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<td>0.0002</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

D.R.C. 0.0000 0.0000 0.0000 0.0000

(3a) $R^2$ 0.9889 0.9894 0.9899 0.9902
(3b) $R^2$ 0.8084 0.8170 0.8224 0.8257
(3c) $R^2$ 0.4310 0.4342 0.4341 0.4326
(3d) $R^2$ 0.5032 0.5026 0.5021 0.5018
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